Decentralized Cooperative Conflict Resolution Among Multiple Autonomous Mobile Agents

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***Abstract*— In this paper we consider policies for cooperative, decentralized traffic management among a number of autonomous mobile agents. The conflict resolution problem is addressed considering realistic restrictions on possible maneuvers. We formulate this problem as one in Mixed Integer Linear Programming (MILP). The method, which proves successful in a centralized implementation with a large number of cooperating agents, is also extended to a decentralized setting. Conditions for the existence of conflict avoidance maneuvers for a system of 5 autonomous agents with a transitive information structure are provided, along with the explicit policy to be applied by each agent.**

I. Introduction

In recent years, multi-agent system (MASs) have attracted increasing attention and have been proposed for several applications, such as air traffic mangement, planetary exploration, surveillance etc.. MASs offer many potential advantages with respect to single-agent systems such as speedup in task execution, robustness with respect to failure of one or more agents, and scalability. On the other hand, MASs introduce challenging issues such as the handling of distributed information data, the coordination among agents, the choice of communication protocols, and the design and verification of decentralized control laws [19].

In this paper, we consider the problem of managing the traffic of MASs for which the start and goal configuration of each agent is assigned, and a path has to be decided for each so that any collision between them is avoided. This can be done by using a centralized approach in which safe trajectories for all agents are computed by a unique decision maker (see e.g. [18],[13],[10],[6], [16] for air traffic conflict management). Although correct and complete algorithms for the centralized traffic management problem may exist, they typically require a large amount of computational resources. Furthermore, centralized approaches typically are very prone to faults of the decision maker. Alternatively, decentralized approaches can be adopted, by which each agent plans its own trajectory based only on information limited to neighboring agents. A decentralized approach is typically faster to react to unexpected situations, but safety verification is an issue as domino effects of possible conflicts may prevent convergence to solutions in some conditions.

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Several authors have considerd decentralized control of multiple mobile agents. In [4] authors propose a hybrid control architecture with parallel problem solving which guarantees collision avoidance. In [8] the problem of path planning is divided into global and local path planning, and AI techniques are used in combination with real-time techniques. In [9] and [20], formations of robots are considered, where a motion plan for the overall formation is used to control a single ”lead” robot while the ”followers” are governed by local control laws, sensing their positions relative to neighboring robots. In [7] a framework exploiting the advantages of centralized and decentralized planning for multiple mobile robots with limited ranges of sensing and communication maneuvering in dynamic environments, is presented. In aircraft management system, decentralized conflict resolution schemes, which are often referred to as “free-flight” strategies, are a topic of growing recent interest ([1], [2], [6],[15],[17]).

In this paper, we propose a policy for cooperative, decentralized traffic management among several MASs addressing realistic restrictions on possible maneuvers of the agents. We formulate this problem using Mixed Integer Linear Programming (MILP) techniques. The method builds upon a technique proposed in [16], which proved successful in centralized implementations with large numbers of cooperating agents, and is extended here to a decentralized setting. The strategy is modeled within a hybrid system framework, and safety is studied using tools from the relative theory. A theorem that ensure safety up to the 5 agents case is proven and safety of the hybrid system is verified. In particular, within the theorem conditions for the existence of conflict avoidance maneuvers for a system of up to 5 autonomous agents with a transitive information structure are provided, along with the explicit policy to be applied by each agent.

The paper is organized as follows. In section II centralized and decentralized cooperative control schemes are proposed and relative information structures reported. The N agents decentralized transitive and cooperative scheme is described with more details in section III. In section IV conflict avoidance constraints are described. Finally, in section V sufficient conditions for the safety of the decentralized transitive information structure are provided. In particular the proof of the theorem is provided.

II. Centralization, Decentralization, and Information Structures

Let the configuration of the i–th autonomous mobile agent be described by a point $(x_i, y_i, \theta_i) \in \mathbb{R} \times \mathbb{R} \times S^1$ where x_i, y_i are the coordinates of the center of the i-th agent and θ_i is the direction of motion or *heading angle*. A *conflict* (or *collision*) between agents i and j occurs if for some value of t, the distance $A_{i,j}$ is less than the sum of their *safety radii* R_i, R_j, i.e.,

$$A_{i,j} := \sqrt{(x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2} < R_i + R_j. \quad (1)$$

For the sake of simplicity, and without loss of generality, we will assume henceforth that $R_i = R_j$, $\forall i, j$, and let $d = R_i + R_j$ denote the *safety distance*. Hence, agents are considered as discs centered in x_i, y_i of diameter d. In the following, we suppose that inequality (1) is satisfied pairwise for the initial configuration of all agents.

Maneuvering limitations of most vehicles are such that omnidirectional models are inadequate to realistically approach MASs. Hence, we assume that the kinematic model of the i-th agent is subject to nonholonomic motion constraints and is given by

$$\begin{cases} \dot{x}_i = v_i \cos\theta_i \\ \dot{y}_i = v_i \sin\theta_i \\ \dot{\theta}_i = \omega_i \end{cases} \quad (2)$$

where v_i and ω_i are the linear and angular velocities respectively. We also take in due account a constraint on the maximum curvature of trajectories, or equivalently on the minimum steering radius ρ_i, by considering $\omega_i \leq v_i/\rho_i$.

For computational purposes, the model in (2) will have to be considered in discrete time. In this case, we use

$$\begin{pmatrix} x_i \\ y_i \\ \theta_i \end{pmatrix}^+ = \begin{pmatrix} x_i \\ y_i \\ \theta_i \end{pmatrix} + \begin{pmatrix} \delta_i \cos(\theta_i + p_i) \\ \delta_i \sin(\theta_i + p_i) \\ p_i \end{pmatrix}, \quad (3)$$

where δ_i represents the length of a forward step and p_i the heading angle change taken in a unit sampling time. The curvature limitation is implemented here by imposing an upper bound on possible instantaneous heading angle changes $|p_i| \leq p_b$.

A. Centralized Cooperative Schemes

In a centralized control scheme, positions and directions of motion of all agents moving in a predefined region of the workspace are known by a single *Decision Maker* (DM). All possible conflicts must be solved by the DM by finding admissible controls for each of the N agents in the controlled workspace so as to minimize a given cost function. A cooperative centralized cost function is usually written as a (weighted) sum of individual costs,

$$J = \sum_{1}^{N} L_i,$$

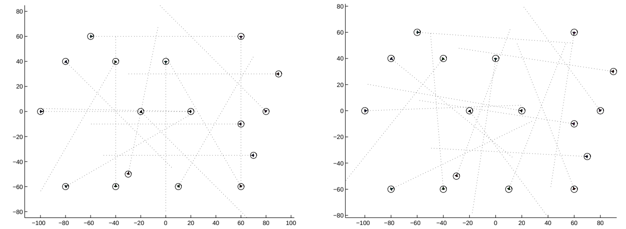

Fig. 1. A traffic management problem for 17 agents (left) can be managed by the centralized MILP-based algorithm (right) in less than 15 sec. on a common workstation.

where L_i may represent e.g. the path length for agent i [6], or the maximum deviation from its nominal direction [16]. Hence, in a centralized cooperative control scheme a single high-dimensional optimal control problem is solved by a single DM (e.g., the *control tower* in traditional airport traffic management systems).

In [14], a centralized cooperative control scheme has been developed for agents with bounded angular velocity as in (2). An optimal nonlinear control problem must be solved with nonlinear constraints given by minimum safety distance conditions. Necessary conditions for optimality have been obtained by applying Pontryagin's Minimum Principle, and an algorithm has been developed to obtain numerical solution of the optimal nonlinear control problem. The complexity of the algorithm grows combinatorially with the number of agents and could handle up to 5 agents in the same workspace ([5], [6], [14]).

Conflict resolution maneuvers using a simplified model, allowing for bounded instantaneous changes of heading angle and velocity, have been considered in [15] and [16]. In this case, maneuvers are obtained as solutions to a mixed-integer linear optimization problem. The algorithm has proven efficient and fast for tens of agents in a common workspace (see fig. 1). The two approaches of [6] and [16] are complementary: the latter addresses large scale problems involving tens of agents moving in relatively large space, while the former represents a smaller scale scenario with few agents, moving by closely knitted trajectories.

B. Decentralized Cooperative Schemes

In decentralized cooperative control schemes, each agent is allowed to take decisions autonomously, based on the information that is available in real time. Several models of decentralized schemes are conceivable, which may differ in the degree of cooperative/competitive behaviour of the agents, and in the information structure [13],[18],[7]. In this paper, we consider cooperative schemes, which can be regarded as instantiations of classical team theory problems (cf. e.g. [11]).

We assume that two agents communicate with each other when and only when their distance is less than a fixed *alert distance*. The size of the alert distance can then be regarded as a degree of centralization/decentralization. Indeed, very large alert distances relative to the maneuvering capabilities

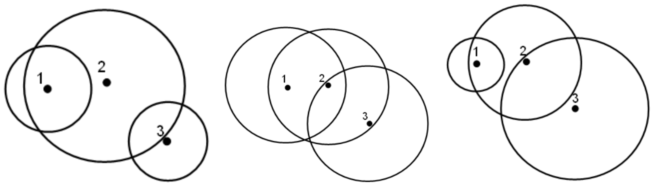

Fig. 2. Several different information structures for three vehicles. Left: $S_1 = \{1\}$, $S_2 = \{1,\ 2\}$, $S_3 = \{3\}$. Middle, non transitive: $S_1 = \{1,\ 2\}$, $S_2 = \{1,\ 2,\ 3\}$, $S_3 = \{2,\ 3\}$; transitive: $S_1 = S_2 = S_3 = \{1,\ 2,\ 3\}$; Right, non transitive: $S_1 = \{1\}$, $S_2 = \{1,\ 2\}$, $S_3 = \{2,\ 3\}$; Right, transitive: $S_1 = \{1\}$, $S_2 = \{1,\ 2\}$, $S_3 = \{1,\ 2,\ 3\}$. Notice that different alert distances cause non reflexivity.

of the agents are tantamount to centralized control, as every agents gets full information on the system while still far away from conflicts. On the other hand, for small alert distances, a miopic resolution policy of one conflict might give raise to a cascade effect on other conflicts, with possibly destabilizing consequences.

A key characteristic of decentralized schemes is the nature of their information structure. Let $S_i(\tau)$ denote the set of indices of agents within distance A_{alert} from the i–th agent at time τ. An information structure is *reflexive* if $i \in S_j \Rightarrow j \in S_i$; it is *transitive* if $i \in S_j$ and $j \in S_k \Rightarrow i \in S_k$. In other words, a MAS is said to have a reflexive information structure if, whenever agent i can get information from agent j, agent j can also get the same *type* of information from agent i. A MAS with reflexive and transitive information structure is such that, whenever agents i and j can exchange information, they do share all the information in their possess. Some illustrative examples are reported in fig. 2.

A cooperative approach to decentralized conflict resolution amounts to assuming that each agent decides its own behaviour based on a policy that tends to optimize a cost function consisting of the sum of individual cost functions extended only to neighbouring agents, i.e.

$$J_i = \sum_{j \in S_i} L_j \tag{4}$$

Notice that the cost 4, along with the agent's dynamics, input and state constraints, define an optimal control problem, which, if well-posed, determines univocally a control policy for agent i. If multiple optimal solutions are possible, a suitable system of rules should be enforced to this purpose.

As a consequence, to each different information structure there corresponds a working mode for the system, i.e. dynamics driven by controls optimizing J_{i,S_i} subject to the non-conflict constraints for all pairs (i, j) with $j \in S_i$.

However when during execution of maneuvers that were planned based on a certain information structure $I = (S_1, \ldots, S_n)$, an agent j with $j \notin S_i$ gets at distance A_{alert} from agent i, the information structure is updated, and optimal paths are replanned according to the new cost function and constraints for agent i. The resulting system is therefore hybrid, as it is comprised of a finite–state machine

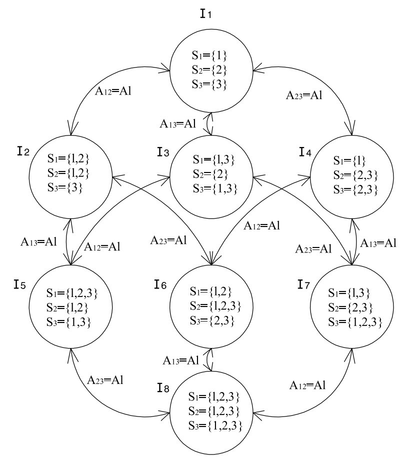

Fig. 3. A decentralized nontransitive scheme with three agents. Each node in the graph corresponds to different costs and constraints in the agents' optimal steering problem. Optimizing controllers for such problems cause different continuous time dynamics at each node. Switching between modes is triggered when an agent enters or exits the alert neighborhood of another.

and of associated continuous-variable dynamic systems, transitions among states being triggered by conditions on the continuous variables.

The big issue with decentralized schemes is obviously that switching among different modes can lead to situations where no feasible solution exist. On the other hand, the decrease in computational complexity of problems solved by each DM allows real-time implementation with embedded controllers, thus introducing a large degree of redundancy which can greatly reduce malfunctioning risks.

To illustrate application of a cooperative decentralized policy on a nontransitive, reflexive information structure, consider a $N = 3$ scenario. There are eight possible states (modes of operation), corresponding to different information structures (see fig. 3). At every state transition, each agent evaluates in real–time the optimal control (heading angle change), from current information structure, for itself as well as for all other agent within its alert radius. Only the control policy evaluated by an agent for itself is then executed, as the one calculated for others may ignore part of the information available to them due to the nontransitivity of the information structure.

Nontransitive schemes tend to amplify both advantages and disadvantages of decentralization. A simulative study reported in [6] has shown the increased robustness of decentralization with respect to failures in the decision making processes. An analytic study of safety of the equivalent hybrid system (for the linear model of (3)) has been presented in [17]. Generalizations to more agents appear to be either overconservative, or complex.

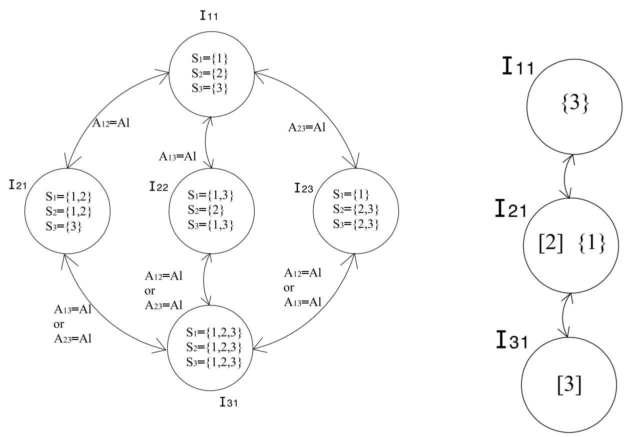

Fig. 4. Left: decentralized transitive scheme with three agents. Notice that nodes I_5, I_6, I_7, I_8 of the nontransitive scheme in fig. 3 coincide here in a single node. Right: the associated relaxed graph, in which only the number of teams and singelton are considered.

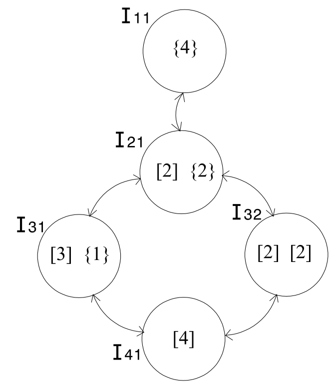

Fig. 5. The decentralized transitive scheme for $N = 4$ agents.

III. Transitive Information Structures for Decentralization

By reflexivity and transitivity of the information structure, whenever agent i appears in S_j, then j also appears in S_i and $S_i = S_j$. Hence, all agents whose indexes are in the same set S_i effectively share the same information and hence execute the same policy. We will therefore refer to $S_i = S_j$ as a “team” in this case. The possible working modes and transitions for the $N = 3$ scenario, under a reflexive and transitive information structure, is illustrated in fig. 4 on the left. Notice the drastically reduced cardinality of the graph nodes.

We describe now the structure of a general N-agents decentralized transitive scheme. Recall that transitions among different operating modes are triggered by zero-crossing conditions for variables of the type $A_{ij}(t) - A_{alert}$. We assume that a minimum dwell time is enforced in each mode, and that no simultaneous transitions are allowed. This assumptione implies for instance that, in fig. 4 on the left, no direct arc exists between state I_{11} and state I_{31}.

To the purposes of safety analysis, a further reduction of the cardinality of modes is instrumental. All nodes in an information graph such as that in fig. 4 on the left, which share the same number of teams and the same number of elements per team, can be identified in a single node as represented in fig. 4 on the right.

A new graph is thus generated, named *relaxed graph*, in which a node is characterized by a list $([n_1], \ldots, [n_m], \{z\})$, where m is the number of non-trivial teams, $n_i > 1$ is the number of elements in the i–th team, and z is the number of trivial (singleton) teams for which $S_j = \{j\}$. For example, nodes I_{21}, I_{22}, I_{23} in fig. 4 are identified in the relaxed graph with a $([2], \{1\})$. Occasionally, nodes of the relaxed graph will be labeled by I_{jk}, where the first index represents the depth of the node in the hybrid system with respect to transitions, while the second index is needed to distinguish nodes of same depth. The first node I_{11} of the hybrid system, for the N agents case, is the one represented by $\{N\}$. This node is thus characterized by $S_i = \{i\}$ for $i = 1, \ldots N$, i.e. all agents are at relative distance larger than the alert distance. From node I_{11} transitions can occur only to node I_{21} that represents a team of two agents and $N - 2$ teams of single agents: $[2]\ \{N - 2\}$.

To the purposes of safety analysis, only transitions from node I_{ik} to I_{jl} with $i \leq j$ are considered. Indeed, inverse transition corresponds to a configuration in which an agent moves at distance larger than the alert distance from each member of the team. In this case, transitions in the relaxed graph involves and modifies only two teams of the starting node. In particular, after a transition two teams are merged in the same team. In the following, we refer to transition form state I_{jk} to state $I_{(j+1)m}$ as j-th *level transition*. Notice that in a N agents scenario there are $N - 1$ levels of transitions in the relaxed graph.

In general, at j-th transition level with $j \leq N$, the nodes are characterized by the following teams and elements: $[a_i]$ for $i = 1, \ldots, k+1$, and $\{N - (j + k)\}$ where $a_i \neq 1$, $k = 0, \ldots, \min\{j - 2, N - j\}$ and $\sum_{i=1}^{k+1} a_i = j + k$.

It is important to notice that even during transitions in the hybrid system, the evolution of agent configurations $(\omega_{ij}(t), \theta_i(t), \theta_j(t), \alpha_{ij})$ is continuous.

In fig. 5 and fig. 6, hybrid systems for $N = 4$ and $N = 5$ are exploited.

IV. Conflict Avoidance Constraints

Our aim is to provide some conditions on the degree of decentralization, i.e. the value of the alert distance, in order to ensure the safety of the decentralized transitive information structure. Safety is referred to the existence of maneuvers that allow agents to avoid possible conflicts for each possible transition in the relaxed graph. In the

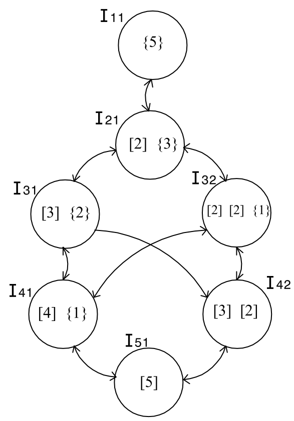

Fig. 6. The decentralized transitive scheme for $N = 5$ agents.

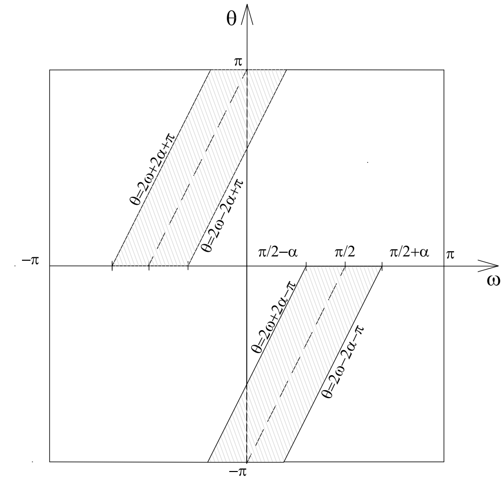

Fig. 7. Unsafe zones: sectors of the (ω, θ) plane for which a conflict is detected.

following, we consider as feasible maneuvers bounded amplitude deviations from the nominal direction of motion.

No-conflict constraints are given by non linear inequalities such as (1). With a geometrical construction and by the introduction of some boolean variables, such nonlinear constraints can be written as linear constraints in the control variables. This construction is reported in details in [17]. In this section only main results are reported for reader convenience.

Consider a general case of n agents in the same team with the dynamics (3). The i-th agent changes its heading angle of a quantity p_i that can be positive (left turn), negative (right turn) or null (no deviation) but anyway bounded by a given value p_b, i.e. $p_i \in [-p_b,\, p_b]$.

For the purpose of safety, the problem is to find an admissible value of p_i for agent i such that all conflicts are avoided with new heading angles $\theta_i + p_i$, for each member of the team. In this section, we formulate no-conflict constraints as inequalities in the unknowns p_i, $\forall i = 1, ..., N$, depending upon agent initial configurations $(x_i, y_i, \theta_i), i = 1, ..., n$. The construction of no-conflict constraints can be done considering agent pairwise and then combining all such conditions for all pairs of agents in the same team.

Given the pair of agents i and j, we define following quantities: $\omega_{ij} = \arctan((y_j - y_i)/(x_j - x_i))$, $A_{ij} = \sqrt{((x_j - x_i)^2 + (y_j - y_i)^2)}$, and $\alpha_{ij} = \arcsin\left(\frac{d}{A_{ij}}\right)$. Let $q_{ij} \overset{def}{=} (\omega_{ij}, \theta_i, \theta_j, \alpha_{ij})$ and p_i, p_j the control variables. The safe set of a system of two agents in configuration $(x_i,\, y_i,\, \theta_i)$ and $(x_j,\, y_j,\, \theta_j)$ is the set of values of p_i and p_j such that $|p_i| \leq p_b$, $|p_j| \leq p_b$ and such that from configurations $(x_i,\, y_i,\, \theta_i + p_i)$, $(x_j,\, y_j,\, \theta_j + p_j)$ no conflict occurs.

Referring to [17] for more details, the safe set for agents i and j can be described by a logical statement $C(q_{ij}, p_i, p_j)$ that is a set of *and* and *or* inequalities, function of q_{ij}, and linear in p_i and p_j. Choosing a linear cost function such as the 1-norm or the ∞-norm of control variables, a Mixed Integer Linear Programming problem must be solved to obtain optimal controls p_i that solve all possible conflicts [15].

The safe set, for the pair (i, j), is thus described by $\Sigma_{c_{ij}} = \{q_{ij} | \exists p_i, p_j \in [-p_b, p_b], C(q_{ij}, p_i, p_j)\}$. Consider a reference system with origin in the position of agent i and direction of x-axis that coincides with the direction of motion θ_i. By studying the equivalent set $\{q_{ij} | \exists p_{ij} \in [-2p_b, 2p_b], C(q_{ij}, 0, p_{ij})\}$, where $p_{ij} = p_i - p_j$, $\omega_{ij} = \omega_{ij} - \theta_j$ and $\theta_{ij} = \theta_i - \theta_j$, the unsafe set represented in the plane $(\omega = \omega_{ij}, \theta = \theta_{ij})$ is reported hased in fig. 7.

Consider now the width Δ_{ij} of the unsafe set band, we have that $\Delta_{ij} = 4\alpha_{ij}$ and decreases with α_{ij}. As a consequence, it decrease as the distance A_{ij} between i and j increase. The value of the bandwidth will be used in the theorem proof.

V. SAFETY OF A DECENTRALIZED N-AGENT SYSTEM

In this section we focus on the safety aspect of the decentralized transitive scheme described in section III. Consider configurations for which a solution of the relative MILP problem exists within each team of agents, we will refer to those as *safe configurations*. In other words, safe configurations are such that no conflict is detected or if a conflict is detected it is solvable with maneuvers of amplitude bounded by p_b. A transition from a state of the hybrid system to another state is a *safe transition* if it starts in a safe configuration and it ends in safe configurations of the new state of the system. Our aim is to compute minimum values of the alert distance to ensure safety for all possible transitions in the hybrid system.

Remark 1: Assume that a minimum alert distance has been computed for the case $N = k$, so that all transition of the associated hybrid system are safe. Consider the case

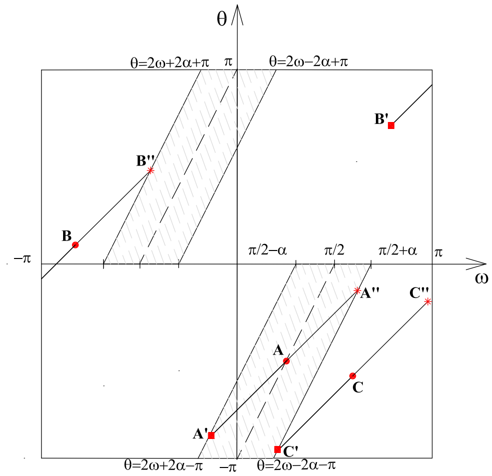

Fig. 8. Case $N = 4$, transition from $[3], \{1\}$ to $[4]$.

$N = k + 1$, all i-th level transitions with $i < k$ are safe (safety conditions on the alert distance have already been obtained in the case $N = k$). For example, based on results obtained for $N = 3$, for case $N = 4$ only transition of type $[3], \{1\} \xrightarrow{T_{31}} [4]$ and $[2], [2] \xrightarrow{T_{22}} [4]$ must be exploited (see fig. 5).

In the following, we propose conflict resolution maneuvers in the worst cases of all transitions in the hybrid systems. Our purpose is to provide *an* admissible maneuver for the worst-case transitions of the hybrid system, thus proving its safety. Optimal maneuvers (with respect to the relative cost function) are computed by agents in the same team by solving the MILP problem described in section IV. We now focus on the proof of the main result of the paper summarized in next theorem.

Theorem 1: Consider N agents with $N \leq 5$ with safety distance d in a common workspace such that initial relative distances are larger than the alert distance A_{alert} or such that they are in a safe configuration of a node of the relaxed graph. Consider the upper bound on possible instantaneous heading angle changes as p_b. If $A_alert \geq d/\sin(p_b/10)$ then each transition that can occur in the hybrid system is safe (i.e. for each transition there exist admissible maneuvers solving conflicts).

Proof: Based on remark 1, we first give conditions on safety for 3 and 4 agents and finally for 5 agents taking into account only 2nd 3rd and 4th level transitions respectively.

Case $N = 3$: in [17] safety has been demonstrated for the decentralized cooperative nontransitive scheme in the $N = 3$ case. The obtained alert distance that ensure safety transitions is $A_{alert} = d/\sin(p_b/4)$ and depends on the safety distance and the bound of admissible controls. A similar demonstration can be applied to the transitive scheme obtaining, for the $N = 3$ case, the same value of the alert distance. Demonstration is omitted for space limitations.

Case $N = 4$: consider the worst case for the first transition, all three agents of $[3]$ (named agents A, B, C) are at the minimum distance A_{alert} with respect to $\{1\}$ (named agent 1), in this case we have $\alpha_{1A} = \alpha_{1B} = \alpha_{1C} = \alpha$. This is a worst case since we have supposed that only two agents (e.g. agent 1 and A) can be at distance A_{alert} at each time t, hence immediately after the transition we have $A_{1B} > A_{alert}$ and $A_{1C} > A_{alert}$. Therefore, $\alpha_{1A} = \alpha > \alpha_{1B}$ and $\alpha > \alpha_{1C}$. Hence, the unsafe sets of pairs $(1, B)$ and $(1, C)$ would be smaller than the unsafe set of $(1, A)$.

Since transition T_{31} starts from a safe configuration, conflicts within team $[3]$ have already been solved. In order to do not generates other conflicts, we don't want agents of team $[3]$ to maneuver. We will assume that the maneuver will be done by agent 1. Let then consider the non collision constraints in coordinates relative to agent 1, (see fig. 8). Assume that, after the transition, agent 1 detect a conflict with agent A, the worst case (reported in fig. 8) is when the minimum maneuver for 1 to avoid the conflict with A generates a conflict with B or C. If a positive deviation is done by agent 1 of amplitude 2α then agents A, B, C will move in configuration A', B', C', while the conflict with A is solved there is a new conflict with C. Otherwise, if a right deviation is done by 1 then agents A, B, C will move in configuration A'', B'', C'', while the conflict with A is solved a conflict with B is detected. For example, let agent 1 to maneuver with $p_1 = 2\alpha$, in order to solve the generated conflict with C another maneuver of amplitude 4α would be needed for agent 1. Hence, in the worst case a singular maneuver of amplitude 6α solves all conflicts. The chosen maneuver is admissible if $6\alpha \leq p_b$, and the transition is safe if $A_{alert} \geq d/\sin(p_b/6)$.

Regarding transition $T_{22} : [2], [2] \rightarrow [4]$, agents of the two teams are named 1, 2 and A, B respectively. The worst case occurs when agent 1 is at distance A_{alert} from A and a conflict occurs, between 1 and A, such that a maneuver of amplitude $\pm 2\alpha_{1A}$ is needed by agent 1 to solve the conflict with A (or by agent A to solve the conflict with 1). In worst case both maneuvers generate conflicts between agents 1 and 2. This happens when agent 2 has $\omega = \pi$ and $\theta \leq 2\alpha_{1A}$, both ω and θ in coordinates relative to agent 1.

If this is the case, we let maneuver agent A, instead of agent 1 of amplitude $2\alpha_{1A}$ or $-2\alpha_{1A}$. In worst case also this two maneuvers are such that a conflict between A and B is generated. This worst case configuration is reported in fig. 9 in the coordinates relative to agent 1. Larger unsafe sets are for agent 2 ($\alpha_{12} = \pi/2$) while smaller ones are for A and B agents.

Referring again to fig. 9, a conflict avoidance maneuver will consist in let both 1 and 2 maneuver with amplitude at worst $+6\alpha_{1A}$. With respect to agent 1 this maneuver produce a diagonal displacement for agent A and B and an horizontal displacement of 6α in the (ω, θ) plane without generating other conflicts, see fig. 10. Concluding, in worst

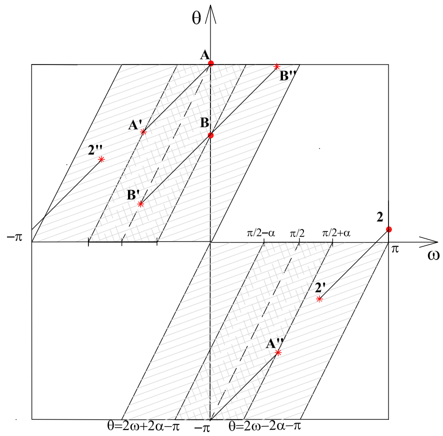

Fig. 9. Case $N = 4$, transition from $[2], [2]$ to $[4]$, in coordinates relative to agent 1.

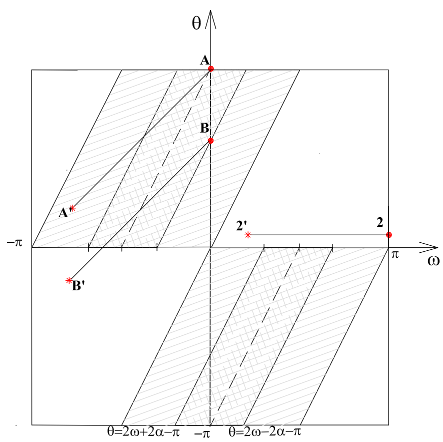

Fig. 10. Case $N = 4$, transition from $[2], [2]$ to $[4]$, in coordinates relative to agent A.

case the transition T_{22} is safe if $A_{alert} \geq d/\sin(p_b/6)$.

Once transitions of $N = 4$ case are safe, regarding $N = 5$ case, transition that need an exploitation are $T_{41} : I_{41} \rightarrow I_{51}$ and $T_{32} : I_{42} \rightarrow I_{51}$, (see fig. 6).

Case $N = 5$: consider the worst case for transition T_{41}, from $[4]\{1\}$ to $[5]$. This is similar to transition T_{31} of the $N = 4$ case, with respect to the $N = 4$ case, in addition there is another agent D at distance $A_{1D} = A_{alert}$. As we have shown previously, with a maneuver of amplitude 4α (for agent 1) conflicts between angent 1 and agents A, B and C are solved. The worst case is when with such maneuver a new conflict between 1 and D is detected. Hence, a total maneuver of amplitude 8α solves all conflict of 1 with A, B, C and D. Concluding, transition is safe if $\alpha < p_b/8$ or equivalently if $A_{alert} \geq d/\sin(p_b/8)$.

Transition T_{32} from $[3][2]$ to $[5]$ is similar to transition T_{22} of the $N = 4$ case. The worst case is the same we reported for T_{22} (see fig. 9), in addition there is another agent C such that once conflicts between 1 and A and B are solved, a new conflict between 1 and C is detected. In order to solve also this confict a total maneuver of amplitude 10α is needed by both 1 and 2. Concluding, transition is safe if $\alpha < p_b/10$ or equivalently if $A_{alert} \geq d/\sin(p_b/10)$.

Concluding, the most restricting condition on the alert distance obtained in the proof is $A_{alert} \geq d/\sin(p_b/10)$ that proves the theorem. ■

To give an idea on the lower bound obtained to the alert distance, if admissible maneuvers are of amplitude smaller than $p_b = 0.35rad$, for $N \leq 5$ agents it is sufficient to impose an initial relative distance larger than the alert distance $A_{alert} \geq 28.6d$ to ensure safety for every transition that can occur. For example if $d = 10cm$ it is sufficient to impose an initial relative distance larger than $3m$.

VI. Conclusions

Centralized and decentralized control schemes for cooperative multiple autonomous agents have been proposed for the conflict resolution problem. In particular, a nontransitive and a transitive decentralized scheme have been considered. For the transitive decentralized scheme, the problem has been described by means of a hybrid system formalism, and its safety has been formally verified for up to 5 agents under the condition that a minimum alert distance between agents is enforced.

Further developments of this work will concern the verification of safety of the decentralized transitive scheme to larger numbers of agents, as well the application to different information structures.

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