

A Real-time Parametric Stiffness Observer for VSA devices

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Abstract— We consider the problem of estimating non-linear time-varying stiffness of a mechanical system based only on force and position measurements. A recent work presented a non-parametric stiffness observer, which converges to within an Uniformly Ultimately Bounded neighborhood of the real stiffness value. The method provides excellent results for applications where the system is persistently excited. In this paper, we provide a parametric identification method that complements the previous solution in that it can provide, after a sufficiently long learning period, a complete model of the nonlinear stiffness, which can be applied henceforth even in the absence of excitation. Convergence conditions for the proposed method are discussed. Simulation and experimental results are provided, illustrating the performance of the proposed algorithm.

I. INTRODUCTION

Stiffness plays a role of paramount importance in many robotic applications, allowing safety [1], dominating interaction control [2], saving energy [3], and preserving mechanisms [4]. As a consequence, a new category of devices is being developed, which goes under the name of Variable Stiffness Actuators (VSA) [5].

From a general viewpoint, a VSA is a mechanical transducer unit presenting a certain input-output characteristic which can be changed with (at least) two degrees of freedom, providing the possibility to regulate both the output rest position and the slope of the output characteristic.

Typical control architectures, as [6], proposed for such actuators aim to attain independent control of link position (or force), and its stiffness with respect to external disturbances. Although effective feedback control schemes using position and force sensors are commonplace in robotics, the problem of controlling stiffness is rather new. Indeed, to the best of our knowledge, in all existing control schemes, a real closed-loop control of stiffness is not possible, because stiffness is not really measured in real-time. Rather, an estimate of the actuator stiffness is inferred from the mathematical model of the actuator. Unfortunately, such practice is prone to errors, as it is extremely sensitive to model inaccuracies, which are typically large for the nonlinear mechanical systems used in VSA.

With the above motivations, in [7] the authors proposed a solution to the problem of measuring stiffness in real-time, proposing a non-parametric observer capable of estimating the non-linear time-varying stiffness of a mechanical system based only on force and position measurements and their (numerical) derivatives. The proposed observer estimation error

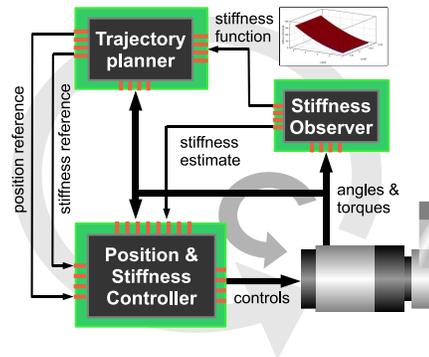


Fig. 1. A two level control system for a Variable Stiffness Robot: On the lowest level a position and stiffness controller tracks the references generated by the higher level trajectory planner. Both control loops rely on knowledge on the stiffness of the actuator: the instantaneous value of the stiffness is used by the inner loop while the shape stiffness function is used by the outer one.

was shown in theory to be Uniformly Ultimately Bounded, and experimental results demonstrated the practicality of the algorithm.

In this work we propose a different method for the estimation of variable stiffness, which tries to overcome some of the limitations of the previous approach. To pursue this goal, the observation problem is projected on the vector basis of a function space. This consists in describing the characteristic output function of the system with a parametric model, whose parameters are updated with an appropriate law to converge to the real stiffness characteristic of the device. The estimate model offers the advantage of being applicable also when the trajectory of the system is not exciting enough, allowing for reconstruction of the stiffness value in a wider set of situations with respect to the non-parametric approach. One further advantage of the parametric structure lies in the possibility to define the model as a function from \mathcal{R}^n to \mathcal{R} , exploiting a wider set of inputs than the non-parametric observer. This, applied to the case of VSA systems, allows to exploit of one additional, usually available, measurement: the internal configuration of the stiffness controlling mechanism.

An advantage of this approach is the possibility to avoid, to a certain extent, the usage of derivatives of the input signals, even if at the cost of convergence speed. A further and stronger advantage offered by parametric observation lies in the possibility to feed the estimate model to higher levels of the control architecture, realizing the control system of Fig. 1. Position-Stiffness trajectory planners can be thought, which using the model information, generate optimized

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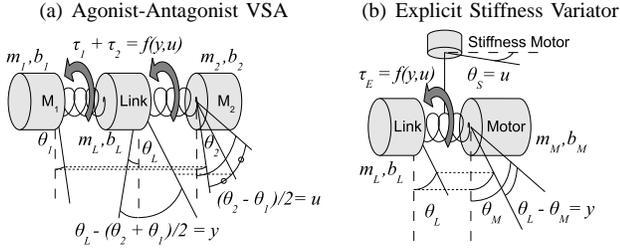


Fig. 2. Two common categories of Variable Stiffness Actuators: the definitions of the deformation angle y , the stiffness angle u and the recoil torque f are shown.

trajectories, guaranteeing safety [1], preserving energy, or pursuing other goals, based on the particular control policy.

Section II states the problem and resumes briefly the solution proposed in [7]. Section III derives the new parametric approach and discusses its properties. Sections IV and V report some results obtained applying the observer on simulated and experimental data-sets, with a comparison between results obtained by both parametric and non-parametric approaches.

II. PROBLEM STATEMENT

Given a generic nonlinear spring whose reaction force f depends on its displacement y and a vector of internal configuration variables u , its stiffness σ can be defined as

$$\sigma(y, u) = \frac{\partial f(y, u)}{\partial y}.$$

An approach for the measurement of non-linear time-varying stiffness of a mechanical system was recently proposed in [7]. It consists in a non-parametric observer which based on the sensed values of torque (or force) and deformation, estimates the stiffness with an Uniformly Ultimately Bounded error defined by

$$|\tilde{\sigma}| > \left| \frac{\partial f}{\partial u} \right| \frac{|\dot{u}|}{|\dot{y}|} + \frac{|\dot{\sigma}|}{\alpha |\dot{y}|} = \frac{1}{\alpha} \left| \frac{\partial \sigma}{\partial y} \right| + \left(\left| \frac{\partial f}{\partial u} \right| + \frac{1}{\alpha} \left| \frac{\partial \sigma}{\partial u} \right| \right) v. \quad (1)$$

In equation 1, the letter v is an upper bound of the ratio between the rates of change of $u(t)$ and the measured deformation $y(t)$, namely $\forall t: v > |\dot{u}(t)|/|\dot{y}(t)|$.

Equation 1 defines the limitations of the non-parametric observation approach. The first limitation, somehow intrinsic to the observation problem, is due to fact that the algorithm uses movement of the device to estimate its stiffness, so it cannot estimate when the movement speed is null.

The second limitation to the non parametric approach comes from the dependency of the characteristic function f on the variable u . In the worst conditions, if the stiffness is changing while outside of observability conditions, the change can not be tracked by the non-parametric observer, which simply stops observing.

A smarter approach to the observation problem should try to exploit the knowledge of the variable u : Its value is not accessible in every situations, as an example consider

the measurements in biological systems that were also considered in [7]. However access to this value is, in variable stiffness actuators, taken for granted. Most of VSA, in fact, can be described as mechanical systems with 3 degrees of freedom: one for rest position of the output shaft, one for the configuration of the stiffness adjusting mechanism (u) and one for the deflection of the output shaft from its rest position (y)¹. Values of those coordinates, or of an equivalent basis, are always measured for feedback control, thus their knowledge should be used to improve estimation performance.

III. PARAMETRIC APPROACH

We propose now a method for circumventing some of the limitations just mentioned, namely by the adoption of a parametric observer which tries to reconstruct the whole force function $f(y, u)$.

Without loss of generality, group the two inputs of the force/displacement characteristic, y and u in a vector x , as in $f(y, u) = f(x)$. Assume that it is possible to write down the elastic force expression on a series expansion on the vector basis of function space, defined on functions from \mathcal{R}^2 to \mathcal{R} , we get²

$$f(x) = \sum_{i=1}^{\infty} f_i(x) c_i, \quad (2)$$

where c_i are constant parameters.

The above expression can be truncated to the N^{th} term, obtaining

$$f(x) = \sum_{i=1}^N f_i(x) c_i + f_r(x) = c^T F(x) + f_r(x) \quad (3)$$

c and $f(x)$ are column vectors of length N , and $f_r(x)$ is the residual term, neglected with the truncation.

The partial derivatives of $f(x)$ with respect to the elements of the vector x are collected in the row-vector

$$\Sigma = \left[\frac{\partial F(x)}{\partial x_i} \right] = \left[\sigma \left| \frac{\partial F(x)}{\partial u} \right. \right]. \quad (4)$$

Exploiting the structure given to the function $f(x)$ by equation 3, the above can be rendered as

$$\Sigma = c^T S + \Sigma_r(x), \quad (5)$$

where $S = S(x)$ is a matrix with the derivatives of the elements of the vector f with respect to the elements of x :

$$S = \left[\frac{\partial f_i(x)}{\partial x_j} \right] = \left[\frac{\partial f_i(x)}{\partial y} \left| \frac{\partial f_i(x)}{\partial u} \right. \right] = [S_1 | S_2]. \quad (6)$$

The output stiffness to be estimated is one of the partial derivatives contained in Σ , thus if an approximation \hat{c} of the

¹Figure 2 shows the definition of these angles on two of the most common Variable Stiffness designs: an Agonist-Antagonist VSA and an Explicit Stiffness Variator.

²the limitation to \mathcal{R}^2 is just practical for our case, but most of the conclusion drawn in this work can be generalized to functions with domain in \mathcal{R}^n .

vector c is known, and the residual term $\sigma_r = \frac{\partial f_r}{\partial y}$ is small enough, stiffness can be approximated as

$$\sigma = \Sigma_1 \approx \tilde{c}^T S_1 = \hat{\sigma}. \quad (7)$$

To discuss the dynamics of the stiffness estimate, we analyze the Lyapunov function and derivative

$$V_{\tilde{\Sigma}} = \tilde{\Sigma} \tilde{\Sigma}^T \Rightarrow \dot{V}_{\tilde{\Sigma}} = \tilde{\Sigma} \dot{\tilde{\Sigma}}^T + \dot{\tilde{\Sigma}} \tilde{\Sigma}^T$$

where $\tilde{\Sigma}$ is defined as the estimation error on Σ . Regarding the derivative $\dot{\tilde{\Sigma}}$, it holds

$$\begin{aligned} \dot{\tilde{\Sigma}} &= \dot{\Sigma} - \dot{\hat{\Sigma}} = \dot{\Sigma}_r + c^T \dot{S} - \dot{\tilde{c}}^T S - \tilde{c}^T \dot{S} = \\ &= \dot{\Sigma}_r + \tilde{c}^T \dot{S} - \tilde{c}^T S. \end{aligned} \quad (8)$$

Choose

$$\dot{\tilde{c}} \triangleq S(S^T S)^{-1} A \operatorname{sgn}(\dot{x}) \tilde{f} \quad (9)$$

where, here, the operation $\operatorname{sgn}(\dot{x})$ is intended component wise, A is a positive definite gain matrix and \tilde{f} is defined as

$$\begin{aligned} \tilde{f} &\triangleq \dot{f} - \tilde{f} = c^T S \dot{x} + \Sigma_r \dot{x} - \tilde{c}^T S \dot{x} = \\ &= \tilde{c}^T S \dot{x} + \Sigma_r \dot{x}. \end{aligned} \quad (10)$$

This implies

$$\begin{aligned} \dot{\tilde{c}} &= S(S^T S)^{-1} A \operatorname{sgn}(\dot{x}) \dot{x}^T (S^T \tilde{c} + \Sigma_r^T) = \\ &= S(S^T S)^{-1} A \operatorname{sgn}(\dot{x}) \dot{x}^T \tilde{\Sigma}^T, \end{aligned}$$

leading to

$$\begin{aligned} \dot{\tilde{\Sigma}} &= \dot{\Sigma}_r + \tilde{c}^T \dot{S} - \left(S(S^T S)^{-1} A \operatorname{sgn}(\dot{x}) \dot{x}^T \tilde{\Sigma}^T \right)^T S \\ &= \dot{\Sigma}_r + \tilde{c}^T \dot{S} - \tilde{\Sigma} \dot{x} \operatorname{sgn}(\dot{x})^T A^T (S^T S)^{-1} S^T S \\ &= \dot{\Sigma}_r + \tilde{c}^T \dot{S} - \tilde{\Sigma} \dot{x} \operatorname{sgn}(\dot{x})^T A^T. \end{aligned} \quad (11)$$

The definiteness of the outer product $P(\dot{x}) = \dot{x} \operatorname{sgn}(\dot{x})^T$, should be discussed. First notice that the matrix, generated by the outer product of two vectors, has all but one of its eigenvalues equal to zero. Being the trace of the matrix equal to the sum of all the eigenvalues, it also equals, in this case, the only non-zero eigenvalue. Looking at the trace of $P(\dot{x})$, it can be easily shown to be

$$\operatorname{trace}(P(\dot{x})) = \sum_i |\dot{x}_i| \geq 0, \quad (12)$$

Implying that the matrix $P(\dot{x})$ is non-negative definite.

Going back to

$$\dot{V}_{\tilde{\Sigma}} = 2 \left(\dot{\Sigma}_r + \tilde{c}^T \dot{S} - \tilde{\Sigma} \dot{x} \operatorname{sgn}(\dot{x})^T A^T \right) \tilde{\Sigma}^T,$$

it is non-positive definite but for the two terms $\dot{\Sigma}_r \tilde{\Sigma}^T$ and $\tilde{c}^T \dot{S} \tilde{\Sigma}^T$. Suppose that the first can be neglected due to negligibility of the residual term, the convergence of the second to zero and the Persistent Excitation of the trajectory of $x(t)$, that is

$$\forall t, \delta t : \alpha_1 I \leq \left(\int_t^{t+\delta t} A \operatorname{sgn}(\dot{x}) \dot{x}^T dt \right) \leq \alpha_2 I \quad (13)$$

will ensure the convergence of the estimate to the real value of stiffness (see [8] for details). If, on the other hand the contribution of the residual term can not be ignored, a bounded error result will be obtained. The discussion of this case is more complicate and is not reported.

To check the convergence of $\tilde{c}^T \dot{S}$ to 0, analyze now the dynamic of the error $\tilde{c} = c - \hat{c}$ with the Lyapunov function, and its Lyapunov derivative:

$$V_c = \tilde{c}^T \tilde{c} \Rightarrow \dot{V}_c = \tilde{c}^T \dot{\tilde{c}} + \dot{\tilde{c}}^T \tilde{c}. \quad (14)$$

Exploiting the equivalent expression of \tilde{f} it holds that

$$\begin{aligned} \dot{\tilde{c}} &= -\dot{\hat{c}} = \\ &= -S(S^T S)^{-1} A \operatorname{sgn}(\dot{x}) \dot{x}^T S^T \tilde{c} + \\ &\quad -S(S^T S)^{-1} A \operatorname{sgn}(\dot{x}) \dot{x}^T \Sigma_r^T \end{aligned} \quad (15)$$

the first of the two terms, namely $SMS^T = S(S^T S)^{-1} A \operatorname{sgn}(\dot{x}) \dot{x}^T S^T$, can be easily shown to be non-negative definite, this because M is the product of

$$\begin{aligned} (S^T S)^{-1} &> 0 \\ A &> 0 \\ \operatorname{sgn}(\dot{x}) \dot{x}^T = P^T(\dot{x}) &\geq 0 \end{aligned}$$

and thus is non-negative definite. Once again, when the second term is negligible, the error on the estimate of c is non-divergent, Persistent Excitation of the trajectory, this time in terms of

$$\begin{aligned} \forall t, \delta t : \\ \alpha_1 I \leq \left(\int_t^{t+\delta t} S(S^T S)^{-1} A \operatorname{sgn}(\dot{x}) \dot{x}^T S^T dt \right) \leq \alpha_2 I \end{aligned} \quad (16)$$

makes the estimate \hat{c} converge to the correct value.

Going back to the evolution of $\tilde{\Sigma}$, it is important to notice that whenever

$$\|\tilde{c}\|_{\dot{S} \dot{S}^T} < \|c\|_{\dot{S} \dot{S}^T}, \quad (17)$$

it yields

$$\|\tilde{c}^T \dot{S}\| < \|c^T \dot{S}\|. \quad (18)$$

Note that the right term $\|c^T \dot{S}\|$ of the last equation the variation induced by \dot{y} and \dot{u} on the stiffness, this is the point where the advantage of the parametric observer over the non parametric one becomes clear. Recall, in fact, that in 1 the error bound is proportional to $\dot{\sigma}$, one of the two elements of $\dot{\Sigma} = c^T \dot{S}$. Equation 18 implies that there exist, in the parametric observer, conditions for which the error bound is smaller than the non-parametric observer error, which are, in substance, those of equation 17.

A. On the speed of convergence

Equation 16 shows, in ultimate analysis, the convergence conditions for the error on the parameters \tilde{c} . These alone are enough to imply the convergence of the stiffness estimate, in fact

$$\lim_{\tilde{c} \rightarrow 0} \tilde{\Sigma} = \lim_{\tilde{c} \rightarrow 0} \tilde{c}^T S = 0. \quad (19)$$

However, all the analysis about the error $\tilde{\Sigma}$ was not pointless: remember that analyzing the dynamics of \tilde{c} and $\tilde{\Sigma}$, we fall in both cases on negative semi-definite Lyapunov derivative function, and must resort to ask Persistent Excitation conditions to ensure convergence of the error. In both situations the error can decrease just along one direction of the error space, but an important difference exists: the dimension of the state-space of \tilde{c} is usually much bigger than that of $\tilde{\Sigma}$ which is just of dimension 2. This consideration leads to state that while the convergence speed of the parameters vector \hat{c} could be slow, convergence of the estimate $\hat{\Sigma}$ will be, in practice, much faster. Experiments and simulations of latter sections will show it is in fact comparable to the speed of the non-parametric observer.

B. Overcoming the need for derivatives

One limitation of the current approach is the need for derivatives of signals x and f . Equation 19 hints a possibility to overcome it, which consists in building an update law which converges \tilde{c} . Such an update law can be built based solely on the prediction error on the estimate of f as follows.

Given an estimate \hat{c} of the vector c , an estimate of the force f can be built as $\hat{f} = \hat{c}^T F$, where $F = F(x)$ as in equation 3, from which the error \tilde{f} for which holds

$$\tilde{f} \triangleq f - \hat{f} = F_r(x) + c^T F - \hat{c}^T F = F_r(x) + \tilde{c}^T F. \quad (20)$$

Defining an update law

$$\dot{\hat{c}}_* \triangleq \alpha(F^T F)^{-1} F \tilde{f}, \quad (21)$$

where B is a positive definite gain matrix (the subscript $*$ is used to distinguish from the update law in 9), yields for the dynamics of the \tilde{c}

$$\begin{aligned} \dot{\tilde{c}} &= -\dot{\hat{c}} = -\alpha(F^T F)^{-1} F (F_r(x) + \tilde{c}^T F) = \\ &= -\alpha(F^T F)^{-1} F F^T \tilde{c} - \alpha(F^T F)^{-1} F F_r(x), \end{aligned}$$

which renders the derivative of the Lyapunov function V_c of equation 14 non-positive definite provided that the truncation error term $F_r(x)$ is negligible. Persistent excitation in terms of

$$\forall t, \delta t : \alpha_1 I \leq \left(\int_t^{t+\delta t} \alpha(F^T F)^{-1} F F^T dt \right) \leq \alpha_2 I \quad (22)$$

will make the estimate \hat{c} converge on the real value c . Convergence of vector \hat{c} yields convergence of the parametric model to the real mechanical characteristic represented by the function $f(x)$; this, by virtue of 19, yields convergence of estimation of stiffness calculated as $\tilde{c}^T S_1$. Nevertheless, deriving the innovation from the error \tilde{f} instead of $\dot{\tilde{f}}$, prevents considerations on convergence speed similar to those derived in subsection III-A. This translates, in practice, in a slower convergence of the estimate of the stiffness value. Given a point x where to measure stiffness, the estimate becomes accurate only after the model has converged in the whole neighborhood x . This renders, at the moment, the derivative-free approach less feasible for real-time measurements like those needed for closed loop control, unless an already

accurate initial guess for the vector c is available. A deeper analysis on this possibility is demanded to future works.

C. On the choice of the Function basis

The discussions of this work prescind from the particular function basis chosen to represent the function $f(y, u)$. The only strict requirements lie on the derivability of the functions composing the basis, such that the matrix S can be derived from the vector f . Nevertheless, a deeper analysis of this aspect of the problem could lead to improvement on the performance of the estimator.

The simplest aspect to consider is that the error dynamic is excited by the derivative of the residual term $f_r(y, u)$, a good choice for the function basis should take this into account. Thus, whenever some information on the shape of the function $f(y, u)$ exists, the design process of the estimator should pick a basis where the function f can be represented exactly by a finite set of basis elements $f_i(y, u)$, or otherwise, try to minimize the approximation error.

Even this aspect of the problem is, in our opinion, deep and unexplored, and needs a more exhaustive analysis, which is demanded to future investigations.

IV. SIMULATION RESULTS

In this section of the paper results of the proposed stiffness observer are presented and compared with the performance of the non-parametric method proposed in [7].

A. Simulation 1

In this first application the parametric observer is applied to estimate the stiffness of an Agonist-Antagonist VSA mechanism (as the one shown in Fig. 1(a)) realized with two identical cubic springs, whose force-displacement characteristic is described by

$$f = (y_i - y_L)^3.$$

This determines a VSA system where the equilibrium point of the link is, in the absence of external loads, in the middle position $y_E = (y_1 + y_2)/2$. The link deflection y , in consequence, is quantified by $y = y_L - y_E$. The configuration of the stiffness can be easily described by completing the configuration space of the mechanism, for example with the variable $u = (y_2 - y_1)/2$. Under these hypotheses the force-displacement characteristic of the system can be easily shown to be $f(y, u) = (2y^3 + 6yu^2)$. As a consequence, the stiffness function is $\sigma(y, u) = 6(y^2 + u^2)$. This particular function can be completely represented on a the function basis of the kind

$$f_k(y, u) = y^i u^j \text{ with } k = \left((i - j)^2 + i + j \right) / 2, \quad (23)$$

using only the first 9 elements of the basis, by $c = [0000000260 \dots]^T$. This ensures that if expressing the function with at least 9 terms of the basis 23 the residual term f_r and its derivatives are null.

The parameters of the two observers are set as follows: for the non-parametric observer, the gain is set to $\alpha = 100$,

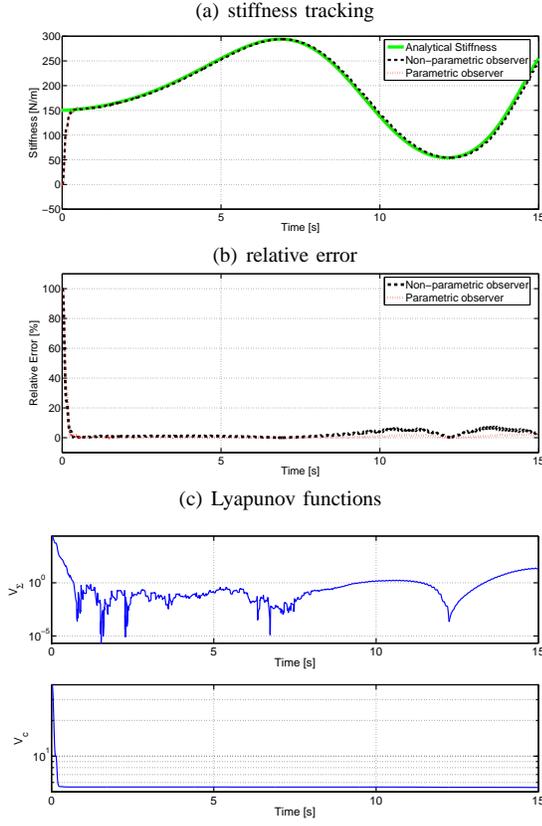


Fig. 3. Results of Simulation 1: performance of the parametric and non-parametric observers tracking the stiffness of an Agonist-Antagonist VSA system featuring a stiffness function which is completely representable on the truncated basis adopted by the parametric observer.

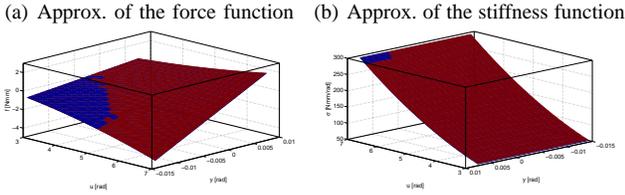


Fig. 4. Results of Simulation 1: observation of the functions describing the recoil force and the stiffness of the system. Blue color is used for the real functions and red color for the estimate ones respectively.

for the parametric one we adopt the first 10 elements of the basis equation 23 and set the gain matrix to $A = 100I$.

Results are shown in Fig. 3: both techniques exhibit analogous satisfactory performance as expected. Panel (c) of Fig. 3 shows the time evolution of the Lyapunov functions V_Σ and V_c on a semi-logarithmic scale: as expected the magnitude of V_c is always non-increasing but after a fast start its convergence speed is sensibly slowed down. The magnitude of V_Σ , on the other hand, is not always decreasing because of the influence of $\tilde{c}\tilde{S}$, but, in practice, it is much faster, shrinking its magnitude by two orders in the first few seconds, and remaining contained afterwards.

Panels on Fig. 4 show a comparison of the reconstructed model in term of the functions \hat{f} and $\hat{\sigma}$ with the real one, highlighting a good conformance between the two.

B. Simulation 2

In this second simulated experiment, both observers are used to track the stiffness of an Agonist-Antagonist VSA, realized with exponential springs. This device, fully described in [7], is characterized by the force and stiffness functions

$$f(y, u) = k(e^{y+u} - e^{u-y}) \Rightarrow \sigma(y, u) = k(e^{y+u} + e^{u-y}). \quad (24)$$

The gain of both observers is kept the same as in the previous simulation, but the state-space of the parametric observer is increased, using up to the 15th element of the function basis of equation 23. This modification is introduced to face the fact that the exponential functions of equation 24 can not be completely represented over a finite sub-set of this basis, and thus, to render the residual term small enough.

Results are shown in Fig. 5. Notwithstanding the imperfect representability of the function over the chosen function basis subset, the parametric observer performance keeps satisfactory.

Another important result is visible in the second simulation, which is an advantage of the parametric observer over the non-parametric one. Looking at the two time intervals $[10, 20]s$ and $[40, 50]s$ in Fig. 4(c), it can be noted that the evolution of y stops: this leads to a drop of the convergence conditions of the non-parametric observer, which simply stops observing. The parametric observer, on the other hand, is building its estimate also on the knowledge of u . Thanks to the model it learned already, it keeps estimating the stiffness of the system even in those adverse conditions. The error is sensibly lower justifies the increased complexity of the algorithm.

V. EXPERIMENTAL RESULTS

A. Set-up description

Finally, to validate the performance of the new parametric observer, ensuring a fair comparison between parametric and non-parametric approaches, the authors chose to apply the observer to the very same data set recorded during the experimental sessions described in [7].

We briefly recall here that the data are relative to an Agonist-Antagonist VSA with exponential springs, similar to that described in section IV-B. Due to uncertainties in the model of the actuator and in the identification of the model parameters, the knowledge of the “true stiffness” is reliable up to an error of about 25%, represented by the horizontal green line in Fig. 5(b) (For full details on the experimental set-up, model identification and data collection, please refer to [7]).

The gain of the non-parametric observer is set to $\alpha = 1$. The parametric observer, uses the function basis adopted in IV-B, while the gain matrix is set to $A = I$. Once again, gains of the two observers are kept comparable for sake of fairness.

B. Results

Results, reported in Fig. 6, show the substantial similarity between the performance of the two methods. Nevertheless, the advantage of the parametric approach, exposed in

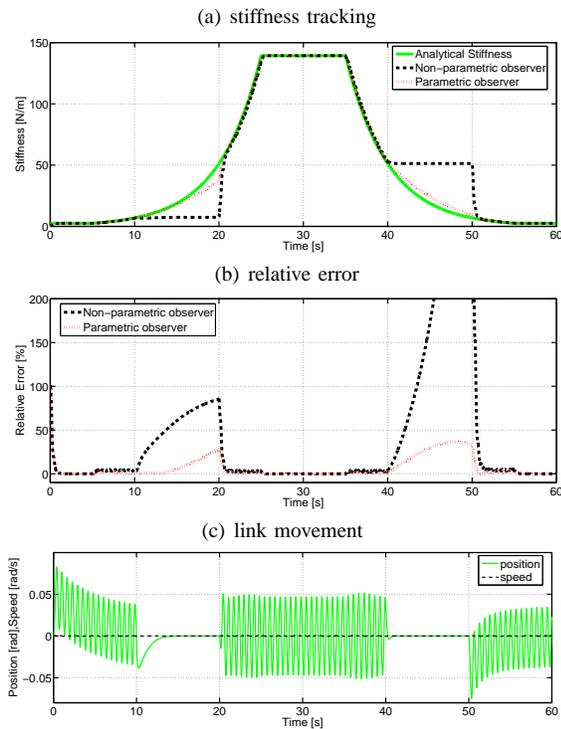


Fig. 5. Results of Simulation 2: performance of the parametric and non-parametric observers tracking the stiffness of an Agonist-Antagonist VSA system featuring a stiffness function which is not completely representable on the truncated basis adopted by the parametric observer.

previous sections, is evident once again: due to the drop of speed of y in conjunction with a tangible change in u after time $t = 24s$, the non-parametric observer suffers for a drop of performance. The parametric observer on the other hand, exploiting the information relative to u , and the collected information about the model, does not suffer from this inconvenience, keeping the relative error small, comparable to the model reliability threshold of 25%.

VI. CONCLUSIONS AND FUTURE WORK

This work presented a parametric observer designed to measure the non-linear, time-varying stiffness of a VSA device, using force and position sensors. The method is an evolution of a non-parametric observer recently presented in literature. At the cost of using a bigger state-space the proposed solution is proven to present two main advantages over the former one: the possibility to use the measurement of the stiffness-setting angle, and the capacity to reconstruct the shape of the stiffness function. Finally, an interesting aspect of the new algorithm, i.e. the possibility not to use the derivative of the input signals, was introduced to be addressed deeply in future works.

Conditions for the convergence of the algorithm were derived and then performance of the observer was compared with results obtained with the former approach, both with simulations and experimental data.

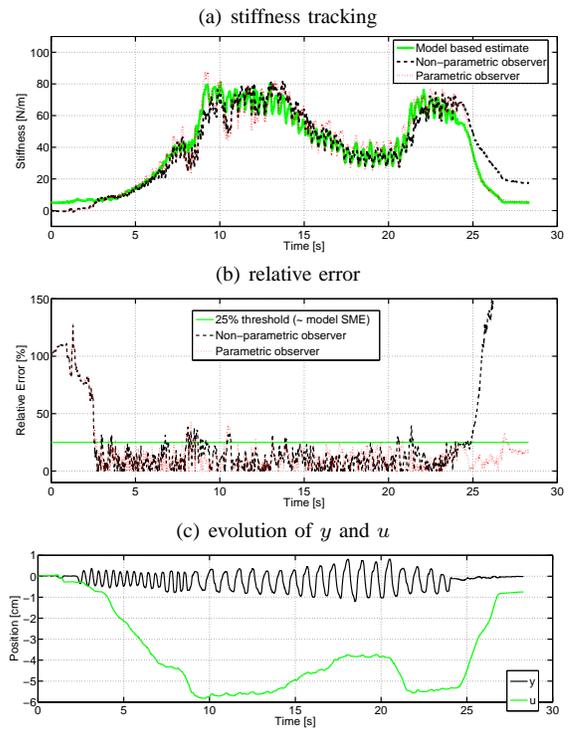


Fig. 6. Experimental Results: results obtained using the parametric and non-parametric algorithm are compared in terms of tracking and relative error on panels a and b, in panel c the evolution of y and u is shown. Notice how the performance of the two observers are similar for most of the experiment but differ (in favor of the parametric observer) after second 24.

VII. ACKNOWLEDGMENTS

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