ACTIVE SUSPENSIONS DECOUPLING BY ALGEBRAIC FEEDBACK

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Active suspensions of advanced vehicles allows the active rejection of external disturbances exerted directly on the sprung mass of the vehicle and due to the road surface irregularity. We focus on the road irregularity disturbances with the purpose of isolating the chassis from vibrations transmitted through suspensions. The paper is aimed at the synthesis of a decoupling control law of the regulated outputs, i.e., roll, pitch and chassis height, from the external disturbances. The framework throughout is the geometric approach to the control of dynamic systems. It will be shown that a controlled and conditioned invariant subspace exists and this allows the perfect disturbance localization by feeding back the suspensions heights.

1 Introduction

Active suspensions are employed in advanced vehicles in order to enhance both ride comfort and safety. The actuation of suspensions along with proper sensor systems allows the vehicles controller to actively reject external disturbances. In most of the conventional cars, rejection of disturbances is obtained by passive devices providing a damping force constraint at all frequencies and generally unable to attenuate both low and high frequency vibrations. On the contrary, active suspensions are able to change the damping force according to the sensed vibrations and can improve the dynamic performance of the whole system. The control of active suspensions has been widely investigated in the literature. Hrovat[5] studied the problem of optimal design of active suspensions by casting it into an equivalent linear–quadratic (LQG)–optimization problem. The problem of estimating suspension parameters was investigated in [8] and [9] where an adaptive observer and an extended Kalman filter were implemented in order to identify parameters.

Two different types of disturbances can influence vehicle dynamics. One acts directly on the sprung mass of the vehicle and can be generated by lateral accelerations, the other type of disturbances is due to road irregularity and is transmitted through the suspensions.

In this paper we focus on the last type of disturbances and our purpose is to isolate the chassis from vibrations transmitted through suspensions. The paper is aimed at the synthesis of a decoupling control law making the regulated outputs, i.e. roll, pitch and chassis height, insensitive to the external disturbances.

The framework throughout is the geometric approach to the control of dynamic systems[3, 4, 11]. It will be shown that the regulated variables can be decoupled from external disturbances. The perfect localization of unaccesible disturbances will be presented as a structural property for vehicles with active suspensions.

A particular attention is devoted to the controller design, it will be proved that a controlled and conditioned invariant subspace exists which allows the perfect disturbance localization by means of an algebraic feedback of the sensors measurements represented by the suspensions heights and velocities.

2 Dynamic model of the vehicle

The mechanical structures of the vehicle in two and three dimensions are reported in fig. 1 a) and in fig. 1 a) and b), respectively. The 3D (2D) vehicle consists of a rigid chassis and two (one) rigid axes. The sprung mass is linked with these axes by means of four (two) passive suspensions and actuators. An independent control action is exerted at each corner of the vehicle. The controlled vertical force u_j, $j = 1, \dots, 4$ ($j = 1, 2$) is generated at the expense of additional energy source such as compressors or pumps. As the aim of the paper is to analyze the structural properties of vehicle mechanisms, the actuator dynamics is not taken into account.

Assume that the vehicle is in an equilibrium configuration (see fig. 1 for illustration) and that the roll center and the gravity center coincides. According to fig. 1–a, let us introduce some notation for the 2D model of the vehicle:

θ_r: variation of the roll angle around the equilibrium;
I_r: moment of inertia of the chassis about the roll axis;
M_b: sprung mass;
z: variation of the height of the M_b center of gravity (CG);
θ_{a1}, I_{a1}: variation angle and inertia of the axis;
z_1, M_{a1}: CG height variation and mass of the axis;
k, β: spring and damping coefficients of suspensions;
k_t, β_t: visco–elastic parameters of tires;
a: half distance between front (or rear) suspensions;

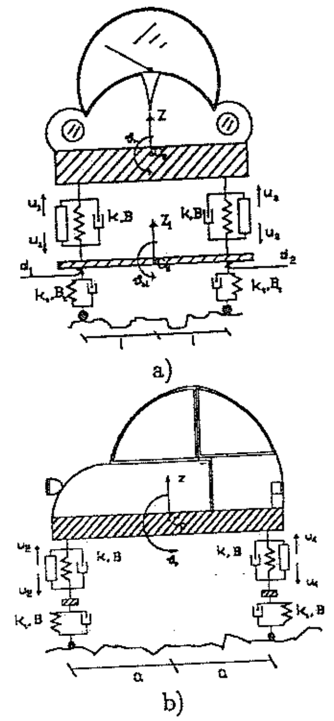

Figure 1: 2D (part a) and 3D (parts a and b) mechanical models of a vehicle with active suspensions.

The 3D model of the vehicle is enriched with the following variables (see fig. 1–b):

θ_{a2}, I_{a2}: variation angle and inertia of the rear axis;
z_2, M_{a2}: height and mass of the rear axis.
l: half distance between of the two axes;
θ_p, I_p: pitch angle and inertia;
d_j: independent, unaccessible, external disturbances exerted on the axes at the j–th wheel.

The 3D model has 7 degrees–of freedom and takes into account the roll (θ_r), the pitch (θ_p) angles of the chassis, the rotations of wheel axes (θ_{a1}, θ_{a2}) and the vertical displacements of the sprung mass (z) and of the two axes (z_{a1}, z_{a2}). Lateral and longitudinal dynamics of the sprung mass are not considered in this 3D model. Note that the reduced 2D model of the vehicle has only 4 dof's (θ_r, z, θ_{a1}, z_1).

Equality of visco–elastic parameters of the passive suspensions has been assumed, hence the dynamics of pitch, roll and vertical motions are decoupled. Such an assumption can be easily satisfied by means of a proper compensating control for the vertical forces u_i's.

Under the assumption that θ_r, θ_p, z, θ_{a1}, θ_{a2}, z_1 and z_2 are small, linear approximation of system dynamics can be considered.

2.1 State space model

A state space representation of the system dynamics is derived for the 2D and the 3D model of vehicle dynamics. Sign conventions for forces, motion and other parameters of vehicle dynamics are defined in fig. 1.

2D dynamic model

We are interested in regulating the chassis posture against disturbances d_1 and d_2 transmitted through the suspensions and generated by road irregularities. Such a type of regulation will be referred to as *ride heights regulation* [10] and consists in controlling the roll and the height of the sprung mass CG. For the 2D case, the 2–dimensional regulated output, the 8–dimensional state, the 2–dimensional input and disturbance vectors are defined respectively as

$$\mathbf{e} = (\theta_r \; z)^T; \tag{1}$$
$$\mathbf{x} = (\mathbf{x}_r^T \; \mathbf{x}_v^T)^T; \tag{2}$$
$$\mathbf{x_r} = (\theta_r \; \theta_{a1} \; \dot{\theta}_r \; \dot{\theta}_{a1})^T;$$
$$\mathbf{x_v} = (z \; z_1 \; \dot{z} \; \dot{z}_1)^T;$$
$$\mathbf{u} = (u_1 \; u_2)^T; \tag{3}$$
$$\mathbf{d} = (d_1 \; d_2)^T. \tag{4}$$

The roll dynamics has been grouped in the vector $\mathbf{x_r}$, while the vector $\mathbf{x_v}$ contains the vertical dynamics.

The state space model of linearized dynamics around the equilibrium configuration is obtained as

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{D}\mathbf{d}; \\ \mathbf{e} = \mathbf{E}\mathbf{x}, \end{cases} \tag{5}$$

where the state matrix is

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{0}_4 \\ \mathbf{0}_4 & \mathbf{A}_{22} \end{bmatrix},$$

with

$$\mathbf{A}_{11} = \begin{bmatrix} \mathbf{0}_2 & \mathbf{I}_2 \\ \mathbf{M_{1k}} & \mathbf{M}_{1\beta} \end{bmatrix}; \quad \mathbf{A}_{22} = \begin{bmatrix} \mathbf{0}_2 & \mathbf{I}_2 \\ \mathbf{M_{2k}} & \mathbf{M}_{2\beta} \end{bmatrix}$$

$$\mathbf{M}_{1k} = \begin{bmatrix} \frac{-2kl^2}{I_r} & \frac{2kl^2}{I_r} \\ \frac{2kl^2}{I_{a1}} & \frac{-2(k_t+k)l^2}{I_{a1}} \end{bmatrix}; \; \mathbf{M}_{1\beta} = \begin{bmatrix} \frac{-2\beta l^2}{I_r} & \frac{2\beta l^2}{I_r} \\ \frac{2\beta l^2}{I_{a1}} & \frac{-2(\beta_t+\beta)l^2}{I_{a1}} \end{bmatrix};$$

$$\mathbf{M}_{2k} = \begin{bmatrix} \frac{-2k}{M_b} & \frac{2k}{M_b} \\ \frac{2k}{M_{a1}} & \frac{-2(k_t+k)}{M_{a1}} \end{bmatrix}; \quad \mathbf{M}_{2\beta} = \begin{bmatrix} \frac{-2\beta}{M_b} & \frac{2\beta}{M_b} \\ \frac{2\beta}{M_{a1}} & \frac{-2(\beta_t+\beta)}{M_{a1}} \end{bmatrix},$$

the input matrix is

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix};$$

with

$$\mathbf{B}_1 = \begin{bmatrix} \mathbf{0}_2 \\ \mathbf{B_{1L}} \end{bmatrix}; \quad \mathbf{B}_2 = \begin{bmatrix} \mathbf{0}_2 \\ \mathbf{B_{2L}} \end{bmatrix};$$

$$\mathbf{B_{1L}} = \begin{bmatrix} \frac{-l}{I_r} & \frac{l}{I_r} \\ \frac{l}{I_{a1}} & -\frac{l}{I_{a1}} \end{bmatrix}; \quad \mathbf{B_{2L}} = \begin{bmatrix} \frac{l}{M_b} & \frac{l}{M_b} \\ \frac{-1}{M_{a1}} & \frac{-1}{M_{a1}} \end{bmatrix},$$

the disturbance matrix is

$$\mathbf{D} = \begin{bmatrix} \mathbf{D_1} \\ \mathbf{D_2} \end{bmatrix};$$

with

$$\mathbf{D_1} = \begin{bmatrix} 0_2 \\ \mathbf{D_{1L}} \end{bmatrix}; \quad \mathbf{D_2} = \begin{bmatrix} 0_2 \\ \mathbf{D_{2L}} \end{bmatrix};$$

$$\mathbf{D}_{1L} = \begin{bmatrix} 0 & 0 \\ \frac{-l}{I_{a1}} & \frac{l}{I_{a1}} \end{bmatrix}; \quad \mathbf{D}_{2L} = \begin{bmatrix} 0 & 0 \\ \frac{1}{M_{a1}} & \frac{1}{M_{a1}} \end{bmatrix},$$

and finally the output matrix of vector (1) is

$$\mathbf{E} = \left[\begin{matrix} 1 \\ 0 \end{matrix} \middle| 0_{(2\times3)} \middle| \begin{matrix} 0 \\ 1 \end{matrix} \middle| 0_{(2\times3)} \right]. \tag{6}$$

3D dynamic model

For the complete 3D model, the controlled output vector is defined as

$$\mathbf{e} = (\theta_r, \ \theta_p, \ z)^T. \tag{7}$$

The 14–dimensional state vector, the 4–dimensional input and disturbance vectors are

$$\mathbf{x} = (\mathbf{x_r}^T \ \mathbf{x_v}^T)^T; \tag{8}$$
$$\mathbf{x_r} = (\theta_r \ \theta_{a1} \ \theta_{a2} \ \dot\theta_r \ \dot\theta_{a1} \ \dot\theta_{a2})^T;$$
$$\mathbf{x_v} = (\theta_p \ z \ z_1 \ z_2 \ \dot\theta_p \ \dot z \ \dot z_1 \ \dot z_2)^T;$$
$$\mathbf{u} = (u_1 \ u_2 \ u_3 \ u_4)^T; \tag{9}$$
$$\mathbf{d} = (d_1 \ d_2 \ d_3 \ d_4)^T \tag{10}$$

and state space linearized dynamics around the equilibrium configuration is given by

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{D}\mathbf{d}; \\ \mathbf{e} = \mathbf{E}\mathbf{x}, \end{cases} \tag{11}$$

where the state matrix is

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_{11}} & 0_{(6\times8)} \\ 0_{(8\times6)} & \mathbf{A_{22}} \end{bmatrix},$$

with

$$\mathbf{A_{11}} = \begin{bmatrix} 0_3 & \mathbf{I_3} \\ \mathbf{M_{1k}} & \mathbf{M_{1\beta}} \end{bmatrix}; \quad \mathbf{A_{22}} = \begin{bmatrix} 0_4 & \mathbf{I_4} \\ \mathbf{M_{2k}} & \mathbf{M_{2\beta}} \end{bmatrix};$$

$$\mathbf{M}_{1k} = \begin{bmatrix} \frac{-4kl^2}{I_r} & \frac{2kl^2}{I_r} & \frac{2kl^2}{I_r} \\ \frac{2kl^2}{I_{a1}} & \frac{-2(k_t+k)l^2}{I_{a1}} & 0 \\ \frac{2kl^2}{I_{a2}} & 0 & \frac{-2(k_t+k)l^2}{I_{a2}} \end{bmatrix};$$

$$\mathbf{M}_{1\beta} = \begin{bmatrix} \frac{-4\beta l^2}{I_r} & \frac{2\beta l^2}{I_r} & \frac{2\beta l^2}{I_r} \\ \frac{2\beta l^2}{I_{a1}} & \frac{-2(\beta_t+\beta)l^2}{I_{a1}} & 0 \\ \frac{2\beta l^2}{I_{a2}} & 0 & \frac{-2(\beta_t+\beta)l^2}{I_{a2}} \end{bmatrix};$$

$$\mathbf{M}_{2k} = \begin{bmatrix} \frac{-4ka^2}{I_p} & 0 & \frac{-2ka}{I_p} & \frac{2ka}{I_p} \\ 0 & \frac{-4k}{M_b} & \frac{2k}{M_b} & \frac{2k}{M_b} \\ -\frac{2ka}{M_{a1}} & \frac{2k}{M_{a1}} & \frac{-2(k_t+k)}{M_{a1}} & 0 \\ \frac{2ka}{M_{a2}} & \frac{2k}{M_{a2}} & 0 & \frac{-2(k_t+k)}{M_{a2}} \end{bmatrix};$$

$$\mathbf{M}_{2\beta} = \begin{bmatrix} \frac{-4\beta a^2}{I_p} & 0 & \frac{-2\beta a}{I_p} & \frac{2\beta a}{I_p} \\ 0 & \frac{-4\beta}{M_b} & \frac{2\beta}{M_b} & \frac{2\beta}{M_b} \\ -\frac{2\beta a}{M_{a1}} & \frac{2\beta}{M_{a1}} & \frac{-2(\beta_t+\beta)}{M_{a1}} & 0 \\ \frac{2\beta a}{M_{a2}} & \frac{2\beta}{M_{a2}} & 0 & \frac{-2(\beta_t+\beta)}{M_{a2}} \end{bmatrix},$$

the input matrix is

$$\mathbf{B} = \begin{bmatrix} \mathbf{B_1} \\ \mathbf{B_2} \end{bmatrix};$$

with

$$\mathbf{B_1} = \begin{bmatrix} 0_{(3\times4)} \\ \mathbf{B_{1L}} \end{bmatrix}; \quad \mathbf{B_2} = \begin{bmatrix} 0_4 \\ \mathbf{B_{2L}} \end{bmatrix};$$

$$\mathbf{B_{1L}} = \begin{bmatrix} \frac{-l}{I_r} & \frac{l}{I_r} & \frac{-l}{I_r} & \frac{l}{I_r} \\ \frac{l}{I_{a1}} & -\frac{l}{I_{a1}} & 0 & 0 \\ 0 & 0 & \frac{l}{I_{a2}} & -\frac{l}{I_{a2}} \end{bmatrix};$$

$$\mathbf{B}_{2L} = \begin{bmatrix} \frac{-a}{I_p} & \frac{-a}{I_p} & \frac{a}{I_p} & \frac{a}{I_p} \\ \frac{l}{M_b} & \frac{l}{M_b} & \frac{l}{M_b} & \frac{l}{M_b} \\ \frac{-1}{M_{a1}} & \frac{-1}{M_{a1}} & 0 & 0 \\ 0 & 0 & \frac{-1}{M_{a2}} & \frac{-1}{M_{a2}} \end{bmatrix},$$

the disturbance matrix is

$$\mathbf{D} = \begin{bmatrix} \mathbf{D_1} \\ \mathbf{D_2} \end{bmatrix};$$

with

$$\mathbf{D_1} = \begin{bmatrix} 0_{(3\times4)} \\ \mathbf{D_{1L}} \end{bmatrix}; \quad \mathbf{D_2} = \begin{bmatrix} 0_4 \\ \mathbf{D_{2L}} \end{bmatrix};$$

$$\mathbf{D}_{1L} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{-l}{I_{a1}} & \frac{l}{I_{a1}} & 0 & 0 \\ 0 & 0 & \frac{-l}{I_{a2}} & \frac{l}{I_{a2}} \end{bmatrix};$$

$$\mathbf{D}_{2L} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{M_{a1}} & \frac{1}{M_{a1}} & 0 & 0 \\ 0 & 0 & \frac{1}{M_{a2}} & \frac{1}{M_{a2}} \end{bmatrix},$$

and finally the output matrix is

$$\mathbf{E} = \left[\begin{matrix} 1 \\ 0_{(2\times1)} \end{matrix} \middle| 0_{(3\times5)} \middle| \begin{matrix} 0_{(1\times2)} \\ \mathbf{I_2} \end{matrix} \middle| 0_{(3\times6)} \right]. \tag{12}$$

3 Localization of disturbances

According to the state space description of vehicle dynamics, the *ride heights regulation* can be rigorously stated as a problem of unaccessible disturbance localization. It consists of synthesizing a state feedback $\mathbf{u} = \mathbf{F}\mathbf{x}$, such that, starting at zero state, the regulated output

$e(t)$ is identically zero for all the admissible disturbances $d(t)$. We attack the problem by using classical tools of the geometric control theory. It is well known[4] that the unaccessible disturbance localization problem has a solution if and only if

$$\mathrm{im}\,(\mathrm{D}) \subseteq \mathcal{V}^*, \tag{13}$$

where $\mathcal{V}^* = \max \mathcal{V}(\mathrm{A}, \mathrm{im}\,(\mathrm{B}), \ker(\mathrm{E}))$ is the maximal (A, B)–controlled invariant contained in $\ker(\mathrm{E})$ and $\mathrm{im}\,(\mathrm{D})$ is the column space of the disturbance matrix. Moreover, for the localization problem to be technically sound, it is to require that the state feedback, other than localizing disturbances in the nullspace of the output matrix, stabilizes the whole system at its equilibrium point.

The following proposition shows that the unaccessible disturbance localization with stability for the regulated output e of the 3D (2D) dynamic system is a structural property of vehicle with active suspensions.

Proposition 1 (Disturbance localization.) *For the 3D [2D] dynamic system in eq.11 [eq. 5] of a vehicle with active suspensions, there always exists a stabilizing state feedback gain* F *which localizes disturbances* d *in the nullspace of the regulated output* $\mathrm{e} = (\theta_r, \theta_p, z)$ $[\mathrm{e} = (\theta_r, z)]$.

Sketch of the proof for 3D case. (For the complete proof the reader is referred to[7].) The proof starts with the definition of the column space of matrix J, included in $\ker(\mathrm{E})$,

$$\mathrm{J} = \begin{bmatrix} \mathrm{J}_1 & 0 \\ 0 & \mathrm{J}_2 \end{bmatrix} \tag{14}$$

where

$$\mathrm{J}_1 = \left[\begin{array}{c|c} \begin{matrix} 0_{(1\times2)} \\ \mathrm{I}_2 \end{matrix} & 0_{(3\times2)} \\ \hline 0_{(3\times2)} & \begin{matrix} 0_{(1\times2)} \\ \mathrm{I}_2 \end{matrix} \end{array}\right]; \quad \mathrm{J}_2 = \left[\begin{array}{c|c} \begin{matrix} 0_2 \\ \mathrm{I}_2 \end{matrix} & 0_{(4\times2)} \\ \hline 0_{(4\times2)} & \begin{matrix} 0_2 \\ \mathrm{I}_2 \end{matrix} \end{array}\right].$$

In[7] it is proven that $\mathrm{im}\,(\mathrm{J})$ is an (A, B)–controlled invariant, thus

$$\mathrm{im}\,(\mathrm{J}) \subseteq \max \mathcal{V}(\mathrm{A}, \mathrm{im}\,(\mathrm{B}), \ker(\mathrm{E})).$$

Since $\mathrm{im}\,(\mathrm{D}) \subseteq \mathrm{im}\,(\mathrm{J})$, it follows that the necessary and sufficient condition for disturbance localization (13) holds. As regards the stability requirement of the controlled couple $(\mathrm{A} + \mathrm{BF},\ \mathrm{B})$, in[7] it is proven that the resolvent $\mathrm{im}\,(\mathrm{J})$ is internally and externally stabilizable. □

Note that for the 2D case the resolvent controlled invariant is given by the column space of

$$\mathrm{J} = \begin{bmatrix} \mathrm{J}_1 & 0 \\ 0 & \mathrm{J}_2 \end{bmatrix}; \quad \mathrm{J}_1 = \mathrm{J}_2 = \left[\begin{array}{c|c} \begin{matrix} 0 \\ 1 \end{matrix} & 0_{(2\times1)} \\ \hline 0_{(2\times1)} & \begin{matrix} 0 \\ 1 \end{matrix} \end{array}\right]. \tag{15}$$

4 Algebraic output feedback

In most real applications, the state is not completely accessible for measurements and the performance of an observer based controller might be unsatisfactory. From an engineering point of view it is interesting to study the localization of disturbance through an algebraic feedback of the sensed outputs.

Assuming that the suspension heights and their time derivatives are accessible for measurements, the output vector

$$\mathrm{y} = [\mathrm{y}_h^T, \dot{\mathrm{y}}_h^T]^T \tag{16}$$

is defined, for the 3D model, through

$$\mathrm{y}_h = \begin{bmatrix} (z - \theta_r l - \theta_p a) - (z_1 - \theta_{a1} l) \\ (z + \theta_r l - \theta_p a) - (z_1 + \theta_{a1} l) \\ (z - \theta_r l + \theta_p a) - (z_2 - \theta_{a2} l) \\ (z + \theta_r l + \theta_p a) - (z_2 + \theta_{a2} l) \end{bmatrix}.$$

Note that for the 2D model of the vehicle, it reduces to

$$\mathrm{y}_h = \begin{bmatrix} (z - \theta_r l) - (z_1 - \theta_{a1} l) \\ (z + \theta_r l) - (z_1 + \theta_{a1} l) \end{bmatrix}.$$

The property of localizing disturbances by means of an algebraic output feedback is formalized in the following.

Proposition 2 *Consider the 3D [2D] vehicle dynamics in eq.11 [eq. 5] with measurement equation* $\mathrm{y} = \mathrm{Cx}$. *There always exists a feedback gain* K *from* y *to* u *which localizes disturbances* d *in the nullspace of the regulated output* $\mathrm{e} = (\theta_r, \theta_p, z)$ $[\mathrm{e} = (\theta_r, z)]$.

Sketch of the proof for the 3D case. It is sufficient to show that the resolvent $\mathrm{im}(\mathrm{J})$, (14), is an (A, C) conditioned invariant, where

$$\mathrm{C} = \left[\begin{array}{cc|cc} \mathrm{C}_H & 0_{(4\times3)} & \mathrm{C}_L & 0_4 \\ \hline 0_{(4\times3)} & \mathrm{C}_H & 0_4 & \mathrm{C}_L \end{array}\right]$$

and

$$\mathrm{C}_H = \begin{bmatrix} -l & l & 0 \\ l & -l & 0 \\ -l & 0 & l \\ l & 0 & -l \end{bmatrix}; \quad \mathrm{C}_L = \begin{bmatrix} -a & 1 & -1 & 0 \\ -a & 1 & -1 & 0 \\ a & 1 & 0 & -1 \\ a & 1 & 0 & -1 \end{bmatrix}.$$

Simply verify that $\mathrm{im}(\mathrm{J}) \cap \ker(\mathrm{C}) = 0$, in fact being

$$\mathrm{CJ} = \begin{bmatrix} \mathrm{C}_{H1} & 0_4 & \mathrm{C}_{L1} & 0_4 \\ 0_4 & \mathrm{C}_{H1} & 0_4 & \mathrm{C}_{L1} \end{bmatrix}$$

with

$$\mathrm{C}_{H1} = \begin{bmatrix} l & 0 \\ -l & 0 \\ 0 & l \\ 0 & -l \end{bmatrix}; \quad \mathrm{C}_{L1} = \begin{bmatrix} -1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & -1 \end{bmatrix},$$

it ensues that $\mathrm{rank}(\mathrm{CJ}) = \mathrm{rank}(\mathrm{J}) = 8$. □

l	$0.9m$	a	$2m$
M_b	$1500kg$	I_r	$360kgm^2$
I_p	$2300kgm^2$	M_{a1}	$40kg$
M_{a2}	$40kg$	I_{a1}	$10.8Kgm^2$
I_{a2}	$10.8Kgm^2$	K	$18E4N/m$
β	$1E3Ns/m$	K_t	$1.96E5N/m$
β_t	$1.92E3Ns/m$		

Table 1: Parameters of vehicle geometry and dynamics; spring and damping coefficients of tires and suspensions.

5 Simulations

A realistic simulation of a road vehicle with active suspensions is here reported to show applications of the disturbance decoupling through algebraic output feedback, for both 2D and 3D cases. The used parameters [6] of the vehicle geometry and dynamics are reported in Table 1. Consider the 2D dynamics (5) with sensed outputs (16). From Proposition 2 the *ride heights regulation* is obtained by the output feedback gain

$$K = 10^4 \begin{bmatrix} 9.5 \; 0.5 \; 0.1056 \; 0.0056 \\ 0.5 \; 9.5 \; 0.0056 \; 0.1056 \end{bmatrix},$$

which localizes disturbances $\mathbf{d}$ in the nullspace of the regulated output $\mathbf{e} = (\theta_r, \mathrm{z})$. Geometrically, the output feedback gain $\mathbf{K}$, makes the resolvent $\mathbf{J}$ (15) invariant in $(\mathbf{A} + \mathbf{BKC})$.

It should be remarked that, for the 2D case, being the triple $(\mathbf{A}, \mathbf{B}, \mathbf{E})$ left–invertible with respect to the input $\mathbf{u}$ and the controlled output $\mathbf{e}$, necessary and sufficient conditions of Theorem 4 in [2], for the algebraic output decoupling feedback, are satisfied. More in detail it can be shown that $\mathcal{V}_m = \mathrm{im}\,(\mathbf{J})$.

As regards the 3D dynamics (11) with measurements $\mathbf{y}$ (16). The output feedback gain for localizing disturbance in the nullspace of the regulated output $\mathbf{e} = (\theta_r, \theta_p, \mathrm{z})$, is given by

$$K = 10^5 \begin{bmatrix} 2.31 & -0.45 & -0.45 & 0.54 & 35\;10^{-4} & -0.01 & -0.01 & 41\;10^{-4} \\ -0.45 & 2.31 & 0.54 & -0.45 & -0.01 & 38\;10^{-4} & 39\;10^{-4} & -97\;10^{-4} \\ -0.45 & 0.54 & 2.3 & -0.46 & -0.01 & 41\;10^{-4} & 8\;10^{-4} & -0.01 \\ 0.54 & -0.45 & -0.47 & 2.30 & 37\;10^{-4} & -97\;10^{-4} & -14\;10^{-3} & 14\;10^{-4} \end{bmatrix}$$

Note that here, according to [2], being the system nor left nor right invertible, only the sufficient condition stated in Property 1 can be checked. Also for the 3D case, $\mathcal{V}_m = \mathrm{im}\,(\mathbf{J})$.

In what follows the influence of the external disturbances, due to road surface irregularities, is simulated with the stabilizing state feedback $\mathbf{F} = -10^3\mathbf{G}$, where

$$G = \begin{bmatrix} 160, & -250, & 88, & -4, & -1.7, & .77, & -440, & 880, & 190, & 10, & -30, & 43, & 1, & -56 \\ -240, & 250, & -88, & 13, & 1.7, & -.77, & -520, & 950, & 190, & 10, & -32, & 44, & 1, & -56 \\ -95, & 88, & -250, & -7, & .77, & -1.7, & 770, & 1300, & 10, & 190, & 36, & 49, & -56, & 1 \\ -170, & -88, & 250, & 10, & -.77, & 1.7, & 700, & 1300, & 10, & 190, & 34, & 50, & -56, & 1 \end{bmatrix}$$

Suppose that the vehicle has a constant speed of $60km/h$ and that the variation of the road surface profile occurs

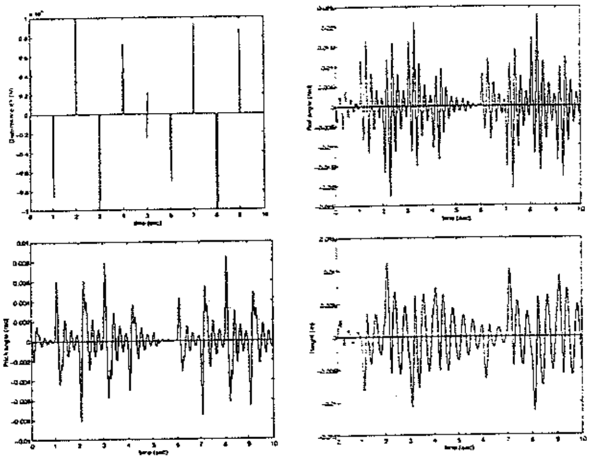

Figure 2: Disturb, roll, pitch angles and vehicle height for a ride of 10 seconds during which the disturbance d_1 and d_3 are exerted on the vehicle. Both outputs for systems with and without disturbance decoupling are reported. Signals identically zero refer to the vehicle with the decoupling feedback.

every $16m$ on the right side of the car ($d_1 \neq 0; d_3 \neq 0; d_2 = d_4 = 0$). Assuming that the front and rear wheels pass the same path, i.e., $d_1 = d(t)$ and $d_3 = d(t - T_c)$ with $T_c = 0.24s$ (first plot in fig. 2), a ride of 10 seconds has been simulated with and without the decoupling feedback

The last three plots in fig. 2 refer to the regulated outputs, roll, pitch angles and vehicle height. The outputs are those relative to both system with and without decoupling. As it is expected, variations of roll and pitch angles and of the vehicle height. due to disturbance $\mathbf{d}$, disappear when the disturbance decoupling gain is fed back. The plots in fig. 3 illustrate the behaviour of signals performed by active suspensions and commanded by the disturbance decoupling controller.

The first three plots in fig. 4 report the behaviour of the regulated outputs roll, pitch angles and vehicle height for the decoupled system when an actuator with saturation level at 1200 N is adopted [6]. The control signal is reported as a function of time in the last plot. From fig. 4 it results that the perfect disturbance decoupling cannot be achieved because of saturation of actuators. But, even in presence of a strong saturation, a considerable reduction of the disturbance is obtained.

6 Conclusions

Localization of external disturbances in road vehicles with active suspensions was investigated. The problem of ride heights regulation, i.e. the regulation of the roll, pitch and vehicle height, was considered. The main result of the paper states that there always exists an algebraic feedback, from the sensed outputs y, able to decouple external disturbances transmitted through suspensions. The aim of this paper is to emphasize that such a decoupling property is a structural property of road vehicles with active suspensions. Moreover it's worthwhile to mention that only axes disturbances d enjoy the decoupling property. In fact it's an easy matter to verify that it's not possible to make the regulated outputs e insensitive to those disturbances which are directly exerted on the sprung mass as, for instance, the lateral accelerations. A case study with a realistic simulation has been reported and some aspects of control implementation were discussed.

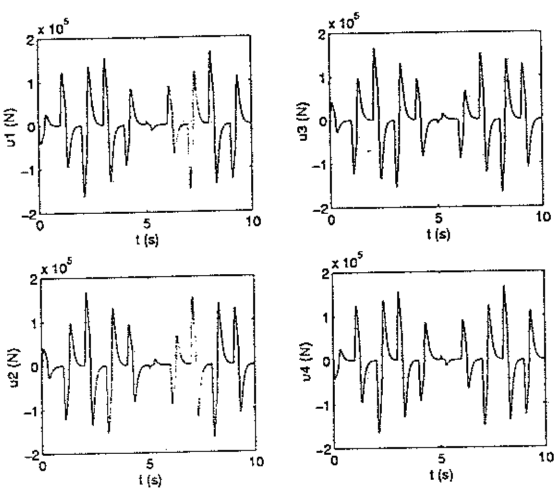

Figure 3: Active suspensions control outputs.

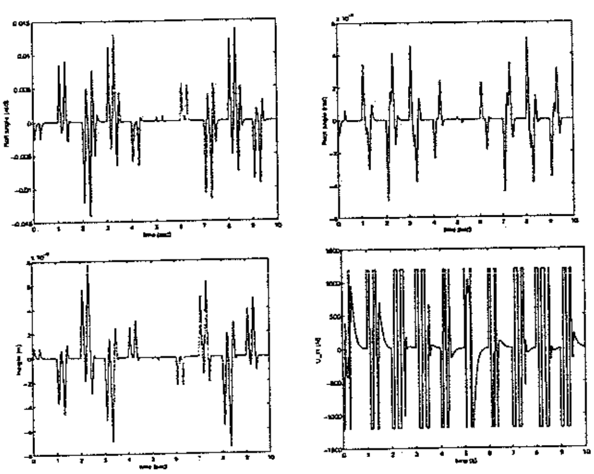

Figure 4: Roll, pitch angles and vehicle height active suspension control behaviour for disturbance decoupling acting through actuators with saturation level at 1200 N.

References

1. Basile G. and Marro G. "L'invarianza rispetto ai disturbi studiata nello spazio degli stati," *Rendiconti della LXX Riunione AAnnuale AEI*, 1969.
2. Marro G. Barbagli F. "The algebraic output feedback in the light of dual-lattice structures," in *Proc. 6th IEEE Mediterranean Conf. on Contr. and Sys.*, Sardinia Italy, June 1998.
3. Basile G. and Marro G. "A state space approach to noninteracting controls, " *Ricerche di Automatica*, vol. 1, n. 1, pp. 68-77, 1970.
4. Basile G. and Marro G. *Controlled and conditioned invariants in linear system theory*, New Jersey, Prentice Hall, 1992.
5. Hrovat D., "Optimal active suspension structures for quarter-car vehicle models," *Automatica*, vol. 26, no. 5 pp. 845-860, 1990.
6. Peng H. and Tomizuka M., "Control of front-wheel-steering rubber tire vehicles," *Report UCB-ITS-PRR-90-5 of PATH programe*, Institute of Transportation Studies, University of California at Berkley, USA, 1990.
7. Prattichizzo D., Mercorelli P., Bicchi A., Vicino A.,"A geometric approach to the control of vehicle with active suspensions," *Internal Report CA-1-397*, University of Siena, 1997.
8. Rajamani R. and Hedrick J.K., "Adaptive observers for active automotive suspensions: theory and experiments," in *IEEE Trans. on Control System Technology*, vol. 3, no. 1, pp. 86-92, March 1995.
9. Ray L. R., " Nonlinear state and tire force extimation for advanced vehicle control," in *IEEE Trans. on Control System Technology*, vol. 3, no. 1, pp. 117-124, March 1995.
10. Unyelioglu K.A., Ozguner U. and Winkelman J., "A decomposition method for the design of active suspension controllers," in *Proc. 13th 1996 IFAC World Congress*, San Francisco USA, 1996.
11. Wonham W.M. and Morse A.S., "Decoupling and pole assignment in linear multivariable systems: a geometric approach," *SIAM J. Control*, vol. 8, no. 1, pp. 1-18, 1970.