needed to fully assess its potentials. Moreover, in the example the superposition facility has not been fully exploited. More complex tasks, involving obstacle avoidance and grasping, are now under investigation to assess the effectiveness of the approach.

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REACHABILITY OF ROLLING PARTS

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The problem of dextrous manipulation and reorientation of polyhedral parts is considered. In this paper we prove a necessary and sufficient controllability–like result, which discloses some of the interesting aspects and perspectives of this problem.

1 Introduction

Manipulating parts and designing mechanism for that purpose is a major problem in robotics. In some cases, the problem is that of reorienting a large number of parts coming in random positions and orientations, to a given posture within assembly tolerances. For such problems, industry most often uses *ad hoc* fixtures, such as vibrating part–feeders, fenced conveyor belts, etc.. In other cases, where the typology of parts is more variate, more flexible manipulation means are preferrable. In highly–flexible automation and robotics, the design of manipulation devices has been attacked by several different approaches, such as by developing dextrous multifingered hands ([Jacobsen *et al.*, 1984], [Salisbury *et al.*, 1985]); using "pushing" or "tilting" actions ([Peshkin and Sanderson, 1988], [Lynch and Mason, 1995]); "regrasping" ([Tournassoud *et al.*, 1987], [Goldberg, 1993]); and "finger gaiting" ([Rus, 1992], [Chen and Burdick, 1993]).

Among these manipulation strategies, those using discontinuous contacts between the manipulator and the part are sometimes regarded as not reliable enough in real–world, unsteady environments. On the other hand, multifingered robot hands are often too costly, heavy, and complex, to be viable in many applications.

The advantage of manipulation by rolling is that it accomplishes dexterity with very simple hardware, while it guarantees that the object is never "left alone" during manipulation. The intrinsic nonholonomic nature of rolling offers many difficulties to the planification and control of such devices, of which only few have been addressed so far.

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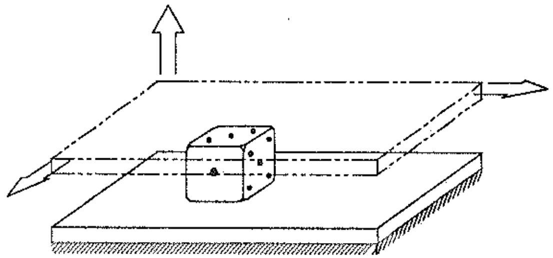

Figure 1: A parallel–jaw gripper can manipulate polyhedral parts

Among the various open problems, the one we start considering in this paper is that of removing the limitation that manipulated objects should have regular surface. The main motivation of such an assumption is that it is rarely verified with industrial parts, which often have edges and vertices.

Again, the simple experiment of rolling a die onto a plane without slipping, and bringing it back after any sufficiently rich path, shows that its orientation has changed in general, and hints to the fact that manipulation of parts with non–smooth (e.g., polyhedral) surface can be advantageously performed by rolling.

Some aspects of graspless manipulation of polyhedral objects by rolling have been considered already in the robotics literature (see e.g. Sawasaki *et al.* [1989], Aiyama *et al.* [1993], Erdmann *et al.* [1991]). However, a complete study on the analysis, planning, and control of rolling manipulation for polyhedral parts is far from being available, and indeed it comprehends many aspects, some of which appear to be non–trivial. In particular, the lack of a differentiable structure on the configuration space of a rolling polyhedron deprives us of most techniques used with regular surfaces. Moreover, peculiar phenomena may happen with polyhedra, which have no direct counterpart with regular objects. In this paper, we start such study by analysing the structure of the set of configurations reachable from a given one.

2 Problem formulation

Consider the simple device depicted in fig. 1, consisting of two plates, one of which is fixed, while the other can translate remaining parallel to the first. A part of known shape is put between the plates, and successively moved by a combination of vertical and horizontal forces at the contacts, that cause it to move. The goal is to bring the part from a given initial configuration to

another desired one. A few considerations are in order:

- as the part is constrained to keep in touch with the two plates, to specify arbitrary desired configurations would require being able to move the lower plate vertically. With no loss of generality we only consider different configurations modulo a rigid translation of the whole mechanism;
- the surface of the part is considered to be piecewise flat, closed, convex and comprised of a finite number of faces, edges, and vertices;
- in general, three motions of a polyhedron on a plane are possible: by sliding on a face, tumbling about an edge, or pivoting about a vertex. But for practical reasons, we assume that only tumbling about an edge is allowed.

The only motions of the parts we will be concerned with are therefore comprised of a sequence of rotations about one of the edges of the face being in contact with the plate, by the amount that exactly brings another face in contact. This action on the parts will be referred to as an elementary tumble, or ET for short.

3 Definitions and first properties

Let $\widetilde{P}$ be a convex polyhedron rolling on a plane P by elementary tumbles. We associate to $\widetilde{P}$ the following sets:

(*a*) $\widetilde{V} = \{v_1, \ldots, v_m\}$ is the set of vertices of $\widetilde{P}$ and $m = \widetilde{V}\#$;

(*b*) $\widetilde{E} = \{e_1, \ldots, e_k\}$ is the set of edges of $\widetilde{P}$ and $k = \widetilde{E}\#$;

(*c*) $\widetilde{F} = \{F_1, \ldots, F_l\}$ is the set of faces of $\widetilde{P}$ and $l = \widetilde{F}\#$.

By the assumption of convexity, parts are topological spheres, hence for their Euler characteristic it holds $\chi = m - k + l = 2$.

The configuration space $\widetilde{M}$ of our problem is the restriction of the space of rigid body configurations $SE(3)$ to those that have one face in contact with the plane P. We give two papameterizations of $\widetilde{M}$ and the first one is as follows. Let Oxy be a fixed reference frame on the plane P. For each face F_i, $1 \le i \le l$, let c_i be the center of gravity of F_i and u_i one of its vertices. Let (x_i, y_i) be the coordinates of c_i, and θ_i be the oriented angle between Ox and $\vec{c_i u_i}$. A configuration of $\widetilde{P}$ on P is uniquely determined by the quadruple (x_i, y_i, θ_i, i), where $i \in \{1, \ldots, l\}$ is the index of the face in contact with P. Then we have

$$\widetilde{M} = \mathbb{R}^2 \times S^1 \times \widetilde{F}. \tag{1}$$

The space $\widetilde{M}$ is endowed with the product metric associated to the metrics of the euclidean space $\mathbb{R}^2$, of the quotient space $S^1 = \mathbb{R}/2\pi\mathbf{Z}$, and of the discrete space $\widetilde{F}$, respectively. The latter is taken to be $\rho(F_i, F_j) = 1 - \delta_{ij}$, the Kronecker symbol. Although very intuitive, this parameterization does not turn out to be the most convenient for our developments. We therefore introduce a slightly more technical description of $\widetilde{M}$ as the set of equivalence classes on a set $\widetilde{M}'$ by the relation $\sim$, where

- the set $\widetilde{M}'$ is defined as the subset of $\mathbb{R}^2 \times \widetilde{V} \times S^1 \times \widetilde{F}$ of points (x, y, v, θ, i) where i is the index of the face F_i in contact with P, v is any of the vertices of F_i (shortly $F_i \ni v$), (x, y) are the coordinates of v and θ is the oriented angle between $\vec{xx'}$ and $\vec{c_i v}$;
- two elements of $\widetilde{M}'$ are equivalent under the relation $\sim$ if $i = i'$ and $\theta' - \theta$ is equal to the oriented angle between $\vec{c_i v}$ and $\vec{c_i v'}$, for any fixed point c_i on face F_i

Note that corresponding to each configuration of the polyhedron, we have an equivalence class with n_{F_i} elements, where n_{F_i} is the number of vertices of the face F_i.

The actions we take on the configurations of the polyhedron are finite sequences of elementary tumbles, referred to as "trips". The length of a trip is the number of ET's it is comprised of. This paper is concerned with the structure induced on the configuration space by trips. We therefore define reachability of a configuration as

Definition 1 *The configuration q_f is reachable from q_0 if there exists a trip steering $\widetilde{P}$ from q_1 to q_2. In this case, we write $q_0 \to q_f$.*

For every $q \in \widetilde{M}$, let $\widetilde{R}_q$ be the reachable set from q, i.e. the set of configurations that can be reached from q in a finite number of ET's.

The structure of the reachable set can be very diverse for different polyhedra. Note first that $\widetilde{R}_q$ is countable by its definition and therefore the inclusion

$$\widetilde{R}_q \subset \widetilde{M}$$

is strict. Introducing the canonical projections

$\Pi_1 : \widetilde{M} \to \mathbb{R}^2;$

$\Pi_2 : \widetilde{M} \to S^1,$

we have that $\Pi_1\left(\widetilde{R}_q\right)$ is trivially infinite and unbounded in $\mathbb{R}^2$. Various possibilities can occur: $\Pi_1\left(\widetilde{R}_q\right)$ (resp. $\Pi_2\left(\widetilde{R}_q\right)$) can be discrete in $\mathbb{R}^2$ (resp. finite in S^1), can have a finite or infinite number of points of accumulation in $\mathbb{R}^2$ (resp. idem in S^1) or can be dense in $\mathbb{R}^2$ (resp. idem in S^1). One can even distinguish differently dense structures for $\widetilde{R}_q$, among which are the following:

a) Density in $\widetilde{M}$:

$$\left(\mathbf{DM}\right) \begin{cases} \forall \varepsilon > 0, \forall q_f \in \widetilde{M}, \\ \exists q' \in \widetilde{R}_q \text{ such that } q' \in B_\varepsilon(q_f) \end{cases}$$

b) Density in $\mathbb{R}^2 \times S^1$ for a given face i:

$$\left(\mathbf{DM}\right)_{\theta,i} \begin{cases} \forall \varepsilon > 0, \forall q_f = (x, y, \theta, i) \in \widetilde{M}, \\ \exists q' \in \widetilde{R}_q \text{ such that} \\ q' \in B_\varepsilon(q_f). \end{cases}$$

c) Density in $\mathbb{R}^2$ for a given vertex v (disregarding the contacting face and its final orientation):

$$\left(\mathbf{DM}\right)_v \begin{cases} \forall \varepsilon > 0, \forall (x, y) \in \mathbb{R}^2, \\ \exists q' = (x', y', v, \theta', i') \in \widetilde{R}_q \text{ such that} \\ (x', y') \in B_\varepsilon(x, y). \end{cases}$$

Here, $\mathbf{B}_\varepsilon(\cdot)$ indicates a ball centered in its argument of radius ε in the suitable metric. Note that

$$\left(\mathbf{DM}\right) \Rightarrow \left(\mathbf{DM}\right)_{\theta,i} \Rightarrow \left(\mathbf{DM}\right)_v.$$

As usual, if the above properties hold for any initial configuration q, the properties will be said to hold globally. In this paper, we explicitly consider the case where the reachable set is dense in $\widetilde{M}$.

The notion of "relative angle" β_v at a vertex will turn out to be fundamental in the rest of this study. For each vertex $v \in \widetilde{V}$, let l_v be its valence, i.e. the number of faces of $\widetilde{P}$ which are adjacent to v, and name such faces as $F_{i_1}, \ldots, F_{i_{l_v}}$. Let α_{i_j}, $1 \le j \le i_{l_v}$, be the angle at v corresponding to face F_{i_j}. The relative angle at v is then defined as

$$\beta_v = 2\pi - \sum_{j=1}^{l_v} \alpha_{i_j}. \tag{2}$$

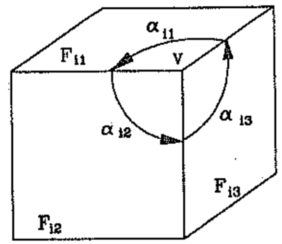

Figure 2: The relative angle at vertex V is defined as $\beta_v = 2\pi - (\alpha_{i1} + \alpha_{i2} + \alpha_{i3})$

The relative angle at a vertex (see fig. 2) is also known in the literature as the curvature concentrated at the vertex. Note that $0 < \beta_v < 2\pi$ since $\widetilde{P}$ is a convex polyhedron with null curvature on its faces. We also have the classical Euler relation given by

Proposition 1 (Euler relation) *Let $\widetilde{P}$ be a convex polyhedron and $\widetilde{V}$ the set of its vertices. Then,*

$$\sum_{v \in \widetilde{V}} \beta_v = 4\pi. \tag{3}$$

Next follow two remarks that are basic properties of the motion of a polyhedra on a plane.

Remark 1. Let $v \in \widetilde{V}$ and suppose that $\widetilde{P}$ rests on P on a face F_i with $F_i \ni v$. By rolling $\widetilde{P}$ on all the faces containing v until coming back to F_i while keeping v immobile, $\widetilde{P}$ is rotated clockwise of an angle β_v around an axis Z_v orthogonal to P and passing through v, i.e., it moves from (x, y, v, θ, i) to $(x, y, v, \theta + \beta_v, i)$. We denote this trip by R_{β_v}, and the analogous anticlockwise trip by $R_{-\beta_v}$. By repeating R_{β_v} clockwise or counterclockwise we can go from (x, y, v, θ, i) to $(x, y, v, \theta + n\beta_v, i)$, $n \in \mathbf{Z}$.

When $\frac{\beta_v}{\pi}$ is irrational, then $\{n\beta_v\}_{n \in \mathbf{Z}}$ is dense in S^1 and the property of reorienting $\widetilde{P}$ "arbitrarily close" (AC for short) to any direction holds.

Remark 2. Suppose that a configuration $q_1 = (x_1, y_1, v, \theta_1, i)$ is steered in $q_1' = (x_1', y_1', v', \theta_1', i')$ by a certain trip T. Then, applying T to any configuration $q = (x, y, v, \theta, i)$, we end up at $q' = (x', y', v', \theta', i')$, where

$$\begin{aligned} (x', y') &= (x_1', y_1') + (x - x_1, y - y_1) \\ &\quad + \Big(\exp\big(i(\theta - \theta_1)\big) - 1\Big)(x_1' - x_1, y_1' - y_1), \end{aligned}$$

$$\theta' = \theta_1' + (\theta - \theta_1).$$

For $i = 1, \ldots, l$, let $\widetilde{M_i} = \mathbb{R}^2 \times S^1 \times \{i\}$, and $\widetilde{T_i}$ be the set of all trips starting and finishing with F_i in contact. For any choice of $m-1$ out of the m vertices of $\widetilde{P}$, labeled as $v_1, \cdots, v_{m-1}$, we have:

Proposition 2 *For every trip $T \in \widetilde{T_i}$, there exist $m-1$ integers $(n_i)_{1 \le i \le m-1}$ such that $\Delta\theta_{|T}$, the total variation of orientation along T is given by:*

$$\Delta\theta_{|T} = \sum_{i=1}^{m-1} n_i \beta_{v_i}.$$

Proof. To each trip $T \in \widetilde{T_i}$ we associate a closed continuous path γ_T defined as follows: let $T = F_i \cdots F_j F_k \cdots F_i$. For all pairs of adjacent faces $F_j F_k$ with common edge e, pick a continuous path γ_{jk} in $\widetilde{P} \backslash \widetilde{V}$ starting from c_j and finishing at c_k, which passes through the edge e only.. The path γ_T is then defined as the concatenation of the γ_{jk} for all pairs of successive faces in T.

The polyhedron $\widetilde{P}$ is topologicaly equivalent to a two-dimensional sphere S^2 and is associated to $\widetilde{P}$, a curvature function K defined as follows ([Spivak, 1979]):

$$K(x) = \begin{cases} 0 & \text{if } x \in \widetilde{P} \backslash \widetilde{V}, \\ \beta_{v_i} & \text{if } x = v_i, 1 \le i \le m. \end{cases}$$

Let $\gamma_1, \cdots, \gamma_{m-1}$ be a homology basis of $\widetilde{P} \backslash \widetilde{V}$ $\left(= S^2 \backslash \widetilde{V}\right)$ (see [Spivak, 1979]). Every path γ_T is therefore homologous to

$$\sum_{i=1}^{m-1} n_i \gamma_i, \quad n_i \in \mathbf{Z}, \ 1 \le i \le m-1.$$

Each γ_i, $1 \le i \le m-1$, is a simple continuous closed curve on $\widetilde{P} \backslash \widetilde{V}$ enclosing only v_i in one of the two connected components it defines. Since any trip T associated with such a γ_i has the same effects on the polyhedron as the trip $R_{\beta_{v_i}}$, we get that the change of orientation along γ_T is equal to (Gauss–Bonnet theorem)

$$\Delta\theta_{|T} = \sum_{i=1}^{m-1} n_i \beta_{v_i}. \tag{4}$$

4 Density of the reachable set

In this section, we state and prove the main result of this paper:

Theorem 1 *For every $q \in \widetilde{P}$, $\widetilde{R}_q$ is globally dense in $\widetilde{M}$ if and only if there exists a vertex $\bar{v}$ such that $\frac{\beta_{\bar{v}}}{\pi}$ is irrational.*

Proof. $\Rightarrow$ The proof of the "if" part is subdivided as follows:

$$\frac{\beta_{\bar{v}}}{\pi} \text{ irrational } \overset{(ii)}{\Rightarrow} \Big(\mathbf{DM}\Big)_{\theta,i_1} \text{ holds for some face } i_1$$
$$\overset{(i)}{\Rightarrow} \Big(\mathbf{DM}\Big) \text{ holds .}$$

Proof of (i): By assumption, there exists a trip that brings the polyhedron AC to (x', y', θ', i_1) for some i_1. Let T be a trip that brings $(x_1, y_1, \theta_1, i_1)$ into (x_2, y_2, θ_2, i), for any fixed i. Since we can go AC to $(x_1, y_1, \theta_1, i_1)$, where

$$\begin{aligned}(x_1, y_1) &= (x', y') + (x_1 - x_2, y_1 - y_2)\\ &\quad -\Big(\exp\Big(i(\theta' - \theta_2)\Big) - 1\Big)(x_2 - x_1, y_2 - y_1),\\ \theta_1 &= \theta' - (\theta_2 - \theta_1),\end{aligned}$$

by remark 2, there exist a concatenation of trips that brings AC to (x', y', θ', i) from (x_2, y_2, θ_2, i), q.e.d.

Proof of (ii): Let $v \in \widetilde{V}$, a vertex different from $\bar{v}$. Suppose that $p = (0, 0, \bar{v}, 0, i_1)$. We can surely reach a point $q_0 = (x_0, y_0, v, \theta_0, i')$ where $\vec{w} = (x_0, y_0) \neq 0$. The trip steering p to q_0 is denoted L and the reverse trip, L^{-1}.

Finally, consider the trip $T_{\beta_{\bar{v}},\beta_v}$ defined as

$$L^{-1} R_{-\beta_v} L R_{-\beta_{\bar{v}}} L^{-1} R_{\beta_v} L R_{\beta_{\bar{v}}}.$$

By remark 2, a simple computation shows that we reach $p_1 = (\vec{t}, \bar{v}, 0, i_1)$, where

$$\vec{t} = 4\sin(\frac{\beta_v}{2})\sin(\frac{\beta_{\bar{v}}}{2})\exp\Big(i\frac{\beta_{\bar{v}} + \beta_v}{2}\Big)\vec{w}. \tag{5}$$

Given $\psi \in S^1$, we can replace in equation (5) and $T_{\beta_{\bar{v}},\beta_v}$ first, $\beta_{\bar{v}}$ by any angle AC to ψ and second, β_v by any of its multiples $n\beta_v$, n integer. In addition, we can easily have the translations $n\vec{t}$, n integer.

Since $\frac{\beta_{\bar{v}}}{\pi}$ is irrational and taking into account the Euler relation, we can suppose that $v \neq \pi$. Therefore, for every integer k, we also have, as in equation (5), the translation

$$\vec{t_k} = 4\sin(\beta_v)\sin(\frac{k\beta_{\bar{v}}}{2})\exp\Big(i\frac{k\beta_{\bar{v}} + 2\beta_v}{2}\Big)\vec{w}, \tag{6}$$

that corresponds to $T_{k\beta_{\bar{v}},2\beta_v}$. We choose k so that $\beta_{k\bar{v}}$ is AC to $\beta_{\bar{v}} - \beta_v$. In this case, $\vec{t'}$ and $\vec{t}$ are AC to be parallel and the ratio of their lenght λ is AC to

$$2\cos(\frac{\beta_v}{2})\frac{\sin(\frac{\beta_{\bar{v}}-\beta_v}{2})}{\sin(\frac{\beta_{\bar{v}}}{2})} = 2\cos^2(\frac{\beta_v}{2}) - \sin(\beta_v)\cot(\frac{\beta_{\bar{v}}}{2}).$$

By remark 1, we can choose $|\lambda|$ as small as we want, which in turn, insures that we can get AC to any point of the line directed by $\exp\Big(i\frac{\beta_{\bar{v}}+\beta_v}{2}\Big)\vec{w}$. Using again remark 1, we can rotate this line and prove $\Big(\mathbf{DM}\Big)_{\bar{v}}$. Furthermore, by remark 1 the polyhedron can be brought AC to $(x, y, \theta, \bar{v}, i_1)$. Therefore we have $\Big(\mathbf{DM}\Big)_{\theta,i_1}$. q.e.d.

$\Leftarrow$ Assume now that there exists a vertex v and a face F_i with $F_i \ni v$ such that $\Big(\mathbf{DM}\Big)_{v,\theta,i}$ holds. We show next (a)$\Rightarrow$(c).

If $\frac{\beta_{v_i}}{\pi}$ is rational for $1 \leq i \leq m$, we have

$$\beta_{v_i} = \frac{p_i}{q_i}\pi, \quad p_i \wedge q_i = 1.$$

Let q be the smallest common multiple of the q_i's. As a consequence of equation (4), the variation of orientation along any γ_T is a entire multiple of $\frac{\pi}{q}$, which is a contradiction with $\Big(\mathbf{DM}\Big)_{v,i,\theta}$. Therefore there must exists $\bar{v} \in \widetilde{V}$ such that $\frac{\beta_{\bar{v}}}{\pi}$ is irrational. This ends the proof of Theorem 1.

5 Conclusions

In this paper we started the study of the structure of the set of configurations that a polyhedron can be brought to reach by rolling on a plane about its edges. The problem appears to be important to practical applications, such as that of automatic part manipulation, as well as theoretically stimulating. Among the many open problems that are left for future work, we mention two of them: the first one concerns a characterization of the reachable set for a general convex polyhedron and a second question deals with defining an efficient algorithm for planning motions of polyhedra by rolling.

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GRASPING ROBUSTNESS MEASUREMENTS: INTEGRATION IN COMPUTER AIDED OPTIMAL DESIGN FOR GRASPING SYSTEMS

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The present contribution fits the topics of computer aided optimal design for robotic grasping systems, with special concern to its application for the choice of a suitable grasp. Speaking of robotic graspng, if it is true that an industry required very crucial demand is to make it as more as possible flexible, special care has to be devoted to the fact that flexibility must be achieved never at expences of stability and robustness. In the present work a technique, to evaluate the robustness of a grasping action is presented. The method which is discussed leads to the introduction of a transmission strengths ellipsoid. Such ellipsoid is conceived to retrieve the axes directions along which an object grasp displays major or minor capability to resist to any external perturbation. To be pointed out that, having to be defined onto a dimensionally non-homogeneus spaces, as the one of forces and torques, such ellipsoid to be properly defined requires an a-priori space homogeneization procedure.

1 Introduction

Grippers are key components in robotized assembly systems. They can represent a significant part of robotized cell cost. Consequently, it is highly wished to increase their flexibility (i.e. their ability to handle all the variants of the workpieces family).

This flexibility in grasp is rarely satisfied by industrial systems and supposes a re-design, or at least an adaptation, of existing grippers. This design operation requires proper consideration of several factors and is basically a complex problem. More precisely, the design process has to take simultaneously into account various constraints, as accessibility to the workpieces, their size and geometry, their mechanical properties, task requirements (surfaces