Proceedings of the 1992 IEEE
International Conference on Robotics and Automation
Nice, France - May 1992

# Mobility and Kinematic Analysis of General Cooperating Robot Systems

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## Abstract

The coordination of the movements of multiple robot arms manipulating a common object is considered. In order to provide a general framework for the study of such cooperating systems as common industrial arms, multifingered hands, legged vehicles, and others, the analysis does not rely on the assumption of full mobility for each cooperating arm, which is otherwise common in related literature. The aim of the paper is to provide a systematic method to characterize the mobility and differential kinematics of general cooperating systems. The proposed analysis and algorithms provide an insight in the structure of the input (joint) - output (task) relationship of such systems.

## 1 Introduction

The exploitation of robotic systems with more complex kinematics than that of conventional, serial-linkage mechanisms is widely perceived as one of the main avenues of development for robotics. An example is the use of multiple arms cooperating in the manipulation of common objects. The concept encompasses systems of different scales and characteristics, such as multifingered hands, multiple arms, and legged vehicles. In most cases, the kinematic analysis of the system is based on the assumption that every single cooperating manipulator has as many degrees-of-freedom as necessary to achieve arbitrary position/orientation in its task space. For example, cooperating arms that rigidly grasp a common object with their end-effectors are usually assumed to have at least six degrees-of-freedom, while the fingers of dextrous hands are supposed to have at least three joints so as to be able to exert arbitrary forces at their fingertips.

This assumption is not always verified in practical applications of cooperating manipulation, as for example in the case that common industrial manipulators with 3 or 4 joints are used. Moreover, the recent introduction of such devices as MIT's Whole-Arm Manipulator (WAM) [Salisbury, 1987], the University of Bologna UB Hand-II [Bonivento, Faldella, and Vassura 1991], as well as the desire for a more general theory of manipulation, renders it necessary to reformulate the problem of mobility and kinematic analysis by dropping the above assumption, and motivates this paper. In fact, the whole-arm manipulation idea implies that also inner links, having deficient kinematic mobility, can contribute to manipulation. Vassura and Bicchi [1990] applied this idea to "whole-hand" manipulation; Mirza and Orin [1990] have shown the advantages of "power grasping" in terms of grip robustness.

The aim of this paper is to provide a systematic method for analyzing the mobility and kinematics of multiple robot systems. The basic questions to which the paper attempts to give an answer are: a) how many parameters are necessary to describe the configurations of a cooperating mechanisms and of its relevant subsystems (mobility analysis), and b) which are the motions that the object can undergo in a given configuration of the mechanisms, and which joint motions will realize them (kinematic analysis). Information on possible free motions of the object in an underconstraining grasp, and on redundant robot joint motions are also provided.

## 2 Modeling of Multiple Robot Systems

The model of the cooperating manipulation system we assume is comprised of an arbitrary number of robot arms (i.e. simple chains of links connected through rotoidal or prismatic joints), and of an object, which is in contact with some or all of the links (see fig.1). Several types of contact models, each of them affecting the motion capabilities of the system in a different way, can be used to describe the interaction between the links and the object. Among the most useful models are probably the point-contact-with-friction model (or "hard-finger"), the "soft-finger" model, and the complete-constraint model (or "very-soft-finger") [Salisbury and Roth, 1982] [Cutkosky, 1985].

We assume that, for the $i$-th of the $n$ contacts, the location of the contact centroid ([Bicchi, Salisbury, and Brock 1990]) $\mathbf{c}_i$ can be measured, for example by means of force/torque-based (intrinsic) contact sensors (as illustrated in the above reference), or any other equivalent tactile sensing device. Fig.1 shows the contact centroid vector ${}^j\mathbf{c}_i$ in the local frame $E_j$ fixed to the $j$-th robot link. The 3-vector $\mathbf{o}_j$ defines the position of the origin of $E_j$ in base frame. Both $\mathbf{o}_j$ and the orientation of $E_j$ w.r.t base frame are known functions of the ma-



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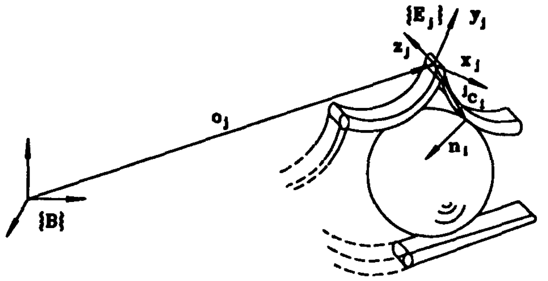


Figure 1: Local and base reference frames in cooperating manipulation.

nipulator(s) geometry and current joint positions (collected in the $r$-vector $\mathbf{q}$). The kinematic constraints imposed by the $i$-th contact can be explicited in terms of the relative velocities of two reference frames ${}^oC_i$ and ${}^mC_i$ both having the origin at the contact centroid $\mathbf{c}_i$, and fixed to the object and to the manipulator link, respectively.

Let $\omega$be the angular velocity of the manipulated object, and $\mathbf{v}$ the linear velocity of a reference point fixed with the object, both expressed in base frame. Choose the object reference point to coincide with the origin of base frame at the instant being considered. The linear and angular velocity (expressed in base frame) of ${}^oC_i$ can be written as

$$ {}^o\dot{\mathbf{c}}_i = \mathbf{v} + \omega \times \mathbf{c}_i, \qquad {}^o\omega_i = \omega, \tag{1}$$

or, for the $n$ contact points, as

$$ {}^o\dot{\mathbf{x}} = \mathbf{G}^T \dot{\mathbf{u}}, \tag{2}$$

where

$$\begin{aligned} \dot{\mathbf{u}} &= (\mathbf{v}^T, \omega^T)^T; \\ {}^o\dot{\mathbf{x}} &= ({}^o\dot{\mathbf{c}}_1^T, \ldots, {}^o\dot{\mathbf{c}}_n^T, {}^o\omega_1^T, \ldots, {}^o\omega_n^T)^T; \\ \mathbf{G} &= \left( \begin{array}{ccc|c} \mathbf{I}_3 & \cdots & \mathbf{I}_3 & \mathbf{O}_{3\times 3n} \\ \mathbf{S}(\mathbf{c}_1) & \cdots & \mathbf{S}(\mathbf{c}_n) & \mathbf{I}_3 \ \cdots \ \mathbf{I}_3 \end{array} \right); \end{aligned} \tag{3}$$

and $\mathbf{S}(\mathbf{c}_i)$ is the cross-product matrix for $\mathbf{c}_i$.

Analogously, the linear and angular velocities of frame ${}^mC_i$ corresponding to joint velocities $\dot{\mathbf{q}}$ can be written in compact form as

$$ {}^m\dot{\mathbf{x}} = \mathbf{D}\dot{\mathbf{q}}, \tag{4}$$

where

$$ {}^m\dot{\mathbf{x}} = ({}^m\dot{\mathbf{c}}_1^T, {}^m\dot{\mathbf{c}}_2^T, \ldots, {}^m\dot{\mathbf{c}}_n^T, {}^m\omega_1^T, {}^m\omega_2^T, \ldots, {}^m\omega_n^T)^T, $$

and $\mathbf{D}$ is a $6n \times r$ matrix whose elements are functions of the robot geometric parameters, joint angles, and contact locations.

Assuming a rigid-body model of the object and the links of the manipulators, the kinematic constraints imposed by contacts can be expressed as

$$ \mathbf{H}\ ({}^o\dot{\mathbf{x}} - {}^m\dot{\mathbf{x}}) = 0 \tag{5}$$

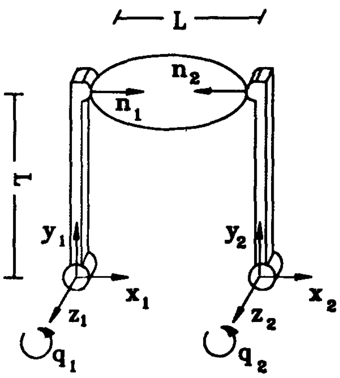


Figure 2: Two one-link manipulators holding an object.

where $\mathbf{H} = \mathbf{I}_{6n}$ (the $6n \times 6n$ iden tity matrix) for complete-constraint contacts, and $\mathbf{H} = (\mathbf{I}_{3n}|\mathbf{O}_{3n\times 3n})$ (where $\mathbf{O}_{j\times k}$ is a $j \times k$ block matrix of zeroes) for hard-finger contacts. For soft finger contacts, we have

$$ \mathbf{H} = \left( \begin{array}{c|ccc} \mathbf{I}_{3n} & & \mathbf{O}_{3n\times 3n} & \\ \hline & \mathbf{n}_1^T & \cdots & 0\,0\,0 \\ \mathbf{O}_{n\times 3n} & \cdots & \cdots & \cdots \\ & 0\,0\,0 & \cdots & \mathbf{n}_n^T \end{array} \right), $$

where $\mathbf{n}_i$ is the normal to the contacting surfaces at the contact centroid. This choice of $\mathbf{H}$ amounts to imposing that relative motions are only allowed which consist of pure rolling, i.e. rotations about axes lying in the tangent plane to contacting surfaces at the contact centroid. An appropriate $\mathbf{H}$ matrix can be easily built even in case the nature of contacts is not homogeneous for different contacts in the same manipulation system.

It should be pointed out that we will not consider explicitly unilateral or conic constraints on contact force in this paper, although such are usually in effect for, e.g., the hard and soft finger contact models. This is because we assume that the system of forces grasping the object is *force closure*, which ensures that grasping forces can always be exerted on the object such that both balance equations and contact force constraints are not violated. An optimal grasp force control scheme that realize grasp stabilization is discussed in [Bicchi, 1992].

### 2.1 Example

To illustrate the procedure used to build up the $\mathbf{G}$, $\mathbf{D}$ and $\mathbf{H}$ matrices, we consider the simple example reported in fig.2. Two one-link arms hold a common object. The links of the manipulators and the distance between the two rotational joints are assumed of the same length $L$. The link frame $E_1$ coincides with the base frame, so that we have $\mathbf{o}_1 = (0\ 0\ 0)^T$; $\mathbf{o}_2 = (L\ 0\ 0)^T$; $\mathbf{z}_1 = \mathbf{z}_2 = (0\ 0\ 1)^T$; $\mathbf{c}_1 = (0\ L\ 0)^T$; $\mathbf{c}_2 = (L\ L\ 0)^T$; $\mathbf{n}_1 = (1\ 0\ 0)^T$; $\mathbf{n}_2 = (-1\ 0\ 0)^T$. The matrices **G** and **D** are evaluated as

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & L & 0 & 0 & L & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -L & 0 & 1 & 0 & 0 & 1 & 0 \\ -L & 0 & 0 & -L & L & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

and

$$\mathbf{D}^T = \begin{pmatrix} -L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

The matrix **H** changes depending on the assumed contact models. For instance, if both contacts are hard-finger, $\mathbf{H} = (\mathbf{I}_6 | \mathbf{O}_{6\times 6})$. If both contacts are soft-finger, the matrix **H** results

$$\mathbf{H} = \left( \begin{array}{c|cccccc} \mathbf{I}_6 & \multicolumn{6}{c}{\mathbf{O}_{6\times 6}} \\ \hline \mathbf{O}_{2\times 6} & 1 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & -1 & 0 & 0 \end{array} \right).$$

If the contact on the first link is soft-finger and hard-finger on the second link, we have

$$\mathbf{H} = \left( \begin{array}{c|cccccc} \mathbf{I}_6 & \multicolumn{6}{c}{\mathbf{O}_{6\times 6}} \\ \hline \mathbf{O}_{1\times 6} & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right),$$

while, for two complete-constraint contacts, we have $\mathbf{H} = \mathbf{I}_{12}$, and so on for different possible combinations.

## 3 Mobility and Kinematic Analysis

The mobility analysis of a cooperating manipulation system consists of the enumeration of the degrees of freedom of the overall mechanism and of its significant subsystems. In particular, we are interested in the evaluation of the connectivity, redundancy and indeterminacy of the system.

The *connectivity* number ($N_c$) of a cooperating mechanisms manipulating a common object can be defined ([Salisbury and Roth, 1982]) as the minimum number of parameters required to specify the position and orientation of the object with respect to the base frame, subject to the kinematic and contact constraints of the system. The *redundancy* number ($N_r$) is defined as the minimum number of parameters required to specify the position and orientation of every link of the mechanism, when considering the object as fixed. The sum of the connectivity and redundancy numbers is the *mobility* ($N_m$) in the strict sense. Finally, the *indeterminacy* number ($N_i$) of a cooperating manipulation system is defined as the minimum number of parameters required to specify the position and orientation of the manipulated object with respect to the base frame, when all joints are locked. With respect to previous results on the subject, the proposed mobility analysis is able to take into account singularities of the mechanism, thus providing exact results whereas the well-known Grübler's formula would only provide inequality relationships. More importantly, the proposed method allows not only the enumeration of the degrees of freedom, but also their kinematic description. It is in fact the aim of the kinematic analysis of a general cooperating robot system to provide the analytic description of its degrees of freedom and their relationships, i.e., to understand which motions can be imparted to the object, and which joint movements can realize them.

Consider for instance the case of a single arm manipulating an object firmly grasped by its end-effector, and suppose that the arm has the minimum number of independent joints necessary to achieve any task-space goal [1]. Obviously, $N_r = N_i = 0$ in this case, while the connectivity of the mechanism is full (equal to the dimension of the task-space). Given any desired velocity of the object $\dot{\mathbf{u}}$, the corresponding joint velocities can be evaluated as

$$\dot{\mathbf{q}} = \mathbf{D}^{-1}\dot{\mathbf{u}}.$$

The case of multiple cooperating arms, each in minimal configuration and firmly grasping the object, is also simple. In fact, in this case the matrix **D** introduced in (4) must be invertible, and we can write

$$\dot{\mathbf{q}} = \mathbf{D}^{-1}\mathbf{G}^T\dot{\mathbf{u}}.$$

To such a system it is possible to apply the so-called master-slave control method for cooperating arms [Nakano, *et. al.*, 1974], consisting in position controlling one of the manipulators (the master), and force controlling the remaining arms. Other, more complex techniques proposed in the field of cooperating arms also apply to this case, see for example [Uchiyama and Dauchez, 1988], [Kokkinis, 1989].

When all the cooperating arms are minimal, except at least one which is redundant, and use only their end-effectors to completely constrain the object, we still have full connectivity and zero indeterminacy, but $N_r > 0$. The problem of finding the joint velocities corresponding to a given object velocity $\dot{\mathbf{u}}$ has multiple solutions. It is customary in the analysis of redundant robotic systems to write all possible solutions as

$$\dot{\mathbf{q}} = \mathbf{D}^{+}\mathbf{G}^T\dot{\mathbf{u}} + \left(\mathbf{I} - \mathbf{D}^{+}\mathbf{D}\right)\mathbf{y}, \qquad (6)$$

where $\mathbf{D}^+$ is the Moore-Penrose pseudo-inverse of **D**, and **y** is a free vector that parameterizes the homogeneous part of the solution. Note that the above relationships only provides a particular solution and a parameterization of possible homogeneous solutions to our problem. The common interpretation that, for $\mathbf{y} = 0$, we obtain the "minimum norm" solution is to be rejected, as discussed in [Melchiorri, 1990].

In the general case addressed in this paper, we have to consider also the case where the system comprises at least some manipulators with deficient kinematics. This may imply that not all arbitrary task-space target velocities can be achieved (the connectivity is less than full), while redundancy can be present ($N_r \geq 0$). Also, on account of considering general contact models, general systems may result indetermined ($N_i \geq 0$). In such a general case, the pseudo-inverse solution can not be applied meaningfully, and we have to turn back to (5) to obtain the correct answer. Substituting (2) and (4) in (5), we have

$$\mathbf{H}\mathbf{G}^T\dot{\mathbf{u}} - \mathbf{H}\mathbf{D}\dot{\mathbf{q}} = \tilde{\mathbf{G}}^T\dot{\mathbf{u}} - \tilde{\mathbf{D}}\dot{\mathbf{q}} = 0.$$

[1] we will term such non-redundant, non-deficient configurations as "minimal"

This relationship can be conveniently put in matrix form as

$$\left(\tilde{\mathbf{G}}^T \ \ -\tilde{\mathbf{D}}\right)\left(\begin{array}{c}\dot{\mathbf{u}}\\ \dot{\mathbf{q}}\end{array}\right) = 0. \qquad (7)$$

Let $\mathbf{Q} = \left(\tilde{\mathbf{G}}^T \ \ -\tilde{\mathbf{D}}\right)$ ($\mathbf{Q}$ is a $6n \times (6+r)$ matrix), and $\mathbf{C}$ a $(6+r) \times q$ matrix whose columns form a basis of the $q$-dimensional nullspace of $\mathbf{Q}$. Finally, partition $\mathbf{C}$ as $\mathbf{C} = \left(\mathbf{C}_1^T \ \mathbf{C}_2^T\right)^T$, where $\mathbf{C}_1$, and $\mathbf{C}_2$ are $6 \times q$, and $r \times q$ blocks, respectively.

The columns of $\mathbf{C}_1$ span the subspace of all possible rigid first-order differential motions of the object that do not break the contact constraints, and the columns of $\mathbf{C}_2$ span the corresponding subspace of joint motions. A complete description of the input-output relationship of a multiple-arm system can be obtained based on the two matrices $\mathbf{C}_1$ and $\mathbf{C}_2$. In fact, by applying column operations only and partitioning appropriately, it is always possible to put the matrix $\mathbf{C}$ in the following form:

$$\left(\begin{array}{c}\mathbf{C}_1\\ \mathbf{C}_2\end{array}\right) = \left(\begin{array}{ccc}\mathbf{C}_{11} & \mathbf{C}_{12} & \mathbf{O}\\ \mathbf{O} & \mathbf{C}_{22} & \mathbf{C}_{23}\end{array}\right) \qquad (8)$$

where the columns of $[\mathbf{C}_{11} \ \mathbf{C}_{12}]$ form a basis of the range space of $\mathbf{C}_1$, $\mathcal{R}(\mathbf{C}_1)$, and the columns of $[\mathbf{C}_{22} \ \mathbf{C}_{23}]$ form a basis of $\mathcal{R}(\mathbf{C}_2)$. Therefore, we have that there exist some coefficient vectors $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ (whose dimensions suit the problem at hand) such that every possible pair of object velocity $\dot{\mathbf{u}}$ and joint velocity $\dot{\mathbf{q}}$ that comply with the kinematic and contact constraints of the multiple arm system can be written as

$$\begin{array}{rcl}\dot{\mathbf{u}} & = & \mathbf{C}_{11}\mathbf{x}_1 + \mathbf{C}_{12}\mathbf{x}_2\\ \dot{\mathbf{q}} & = & \mathbf{C}_{22}\mathbf{x}_2 + \mathbf{C}_{23}\mathbf{x}_3\end{array}. \qquad (9)$$

## 3.1 Discussion

Equation (9) and the structure of the block matrices in (8) contain the desired information on the mobility and kinematics of general cooperating arms. Mobility analysis results can be summarized as follows:

- The mobility of the system is equal to the rank of the $\mathbf{C}$ matrix, i.e. $N_m = \text{rank}(\mathbf{C})$.
- The connectivity of the system is equal to the rank of the $\mathbf{C}_1$ block, $N_c = \text{rank}(\mathbf{C}_1)$ (and hence to the sum of the number of columns of $\mathbf{C}_{11}$ and $\mathbf{C}_{12}$).
- The indeterminacy of the system is equal to the number of columns of $\mathbf{C}_{11}$, $N_i = \text{rank}(\mathbf{C}_{11})$.
- The redundancy of the system is equal to the number of columns of $\mathbf{C}_{23}$, $N_r = \text{rank}(\mathbf{C}_{23})$.

More detailed kinematic information can also be elicited from (8):

- $\mathcal{R}(\mathbf{C}_{11})$ is the indeterminacy subspace of object velocities that are left free by contact constraints. Note that $\mathcal{R}(\mathbf{C}_{11}) = \mathcal{N}(\tilde{\mathbf{G}}^T)$. These object motions cannot be actuated directly by any combination of joint motions. Accordingly, if the object is moving along some direction in this subspace at some instant without external or unmodeled friction forces disturbing it, it will move indefinitely at constant speed in that direction [2].
- $\mathcal{R}(\mathbf{C}_{23})$ is the redundancy subspace of joint velocities that do not affect object velocities, but only modify the configuration of the manipulator arms. Note that $\mathcal{R}(\mathbf{C}_{23}) = \mathcal{N}(\tilde{\mathbf{D}})$.
- If both $\mathbf{C}_{23}$ and $\mathbf{C}_{11}$ result empty ($N_i = N_r = 0$), there is a one-to-one correspondence between joint velocities in $\mathcal{R}(\mathbf{C}_{22})$ and object velociti es in $\mathcal{R}(\mathbf{C}_{12})$, which can be written in parametric form as

$$\left\{\begin{array}{rcl}\dot{\mathbf{q}} & = & \mathbf{C}_{22}\mathbf{x}\\ \dot{\mathbf{u}} & = & \mathbf{C}_{12}\mathbf{x}\end{array}\right. \quad \forall \mathbf{x} \in \Re^{N_c}. \qquad (10)$$

  If the number of columns of the blocks $\mathbf{C}_{12}$ and $\mathbf{C}_{22}$ equals the dimension of the task space, the cooperating system is "minimal". The mapping (10) can be used for further analysis of the kinematic capabilities of the mechanism (e.g., manipulability ellipsoids), or for resolved-rate control of the object motions.

- If $N_i = 0$ and $N_r > 0$, any desired velocity of the object in the feasible subspace $\mathcal{R}(\mathbf{C}_{12})$ can be obtained by means of infinitely many combinations of joint velocities. From (9) we obtain

$$\dot{\mathbf{q}} = \mathbf{C}_{22}\mathbf{C}_{12}^{+}\dot{\mathbf{u}} + \mathbf{C}_{23}\mathbf{y}, \quad \forall \dot{\mathbf{u}} \in \mathcal{R}(\mathbf{C}_{12}), \qquad (11)$$

  where $\mathbf{y} \in \Re^{N_r}$ is a free coefficient vector. Equation (11) presents a particular and parameterised homogeneous solution similar to (6). Any velocity $\dot{\mathbf{u}} \notin \mathcal{R}(\mathbf{C}_{12})$ can not be achieved by the system without breaking contact constraints. Note however that second- or higher-order differential motions in the forbidden directions may still be possible, see e.g. [Nielsen, et. al., 1991].

- If $N_i$ is not zero ($\mathbf{C}_{11}$ is not empty), the object velocity corresponding to a given joint velocity is not uniquely determined by the quasi-static analysis. In fact, from (9),

$$\dot{\mathbf{u}} = \mathbf{C}_{12}\mathbf{C}_{22}^{+}\dot{\mathbf{q}} + \mathbf{C}_{11}\mathbf{y}, \quad \forall \dot{\mathbf{q}} \in \mathcal{R}(\mathbf{C}_{22}), \qquad (12)$$

  where $\mathbf{y} \in \Re^{N_i}$ is a free coefficient vector. The apparent physical non-sense of such indeterminacy is due to the assumed quasi-static model of the system, and can be readily solved by taking into account the object dynamics.

# 4 Case Studies

In the following, two case studies are presented in order to illustrate the application of the above technique. The first case refers to the example of fig.2, i.e.

[2] However, some of these "free" motions can be indirectly controlled in some cases via exploitation of the dynamic couplings the object might have. For a related discussion, see e.g. [Jain and Rodriguez, 1991].

two one-link arms holding a common object. The mobility and kinematic analysis of the system is presented in some different hypotheses about contact constraints. The second example refers to two cooperating SCARA-like robots, together manipulating an object: the cases of complete-constraint contacts and hard-finger contacts are considered.

### 4.1 Case Study 1

Consider the simple example shown in fig. 2, for which the matrices **G**, **D**, and the matrices **H** corresponding to different contact models have been presented previously. The mobility and kinematic analysis of the mechanisms proceeds accordingly.

If both contacts are modeled as hard-finger, at each contact point the object is free to rotate about any direction in the space. Obviously, we expect that some of these rotations will be inhibited by the other contact constraint. In fact, by applying the mobility analysis algorithm, we obtain

$$\mathbf{C}=\begin{pmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{O} & \mathbf{C}_{22} \end{pmatrix}=\left(\begin{array}{cccccc|cc} 0 & 0 & -L & 1 & 0 & 0 & 0 & 0 \\ \hline -L & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right)^T$$

Accordingly, the mobility and connectivity of the system are $N_m = N_c = 2$, being the redundancy $N_r = 0$. The system is quasi-statically indeterminate ($N_i = 1$). In fact, the object may freely rotate, without violating contact constraints, about the axis through the contact points, which is normal to the contact planes: this one-dimensional indeterminacy subspace is analytically described by $\dot{\mathbf{u}}_{indet} = \alpha \mathbf{C}_{11}, \forall \alpha \in \Re^1$.

Besides this free motion, the system may realize a unique coordinated motion consisting of a translation of the object in the $x$ direction, $\dot{\mathbf{u}}_{coop} = \alpha \mathbf{C}_{12}, \forall \alpha \in \Re^1$. Such motion can only be realized by contemporaneously moving the two joints with the same velocity, $\dot{\mathbf{q}}_{coop} = \alpha \mathbf{C}_{22}$.

If both contacts are soft-finger (i.e., rotations about the normal direction to the contact surface are prevented by friction), the indeterminacy of the previous case is eliminated. In fact we have

$$\mathbf{C}=\begin{pmatrix} \mathbf{C}_{12} \\ \mathbf{C}_{22} \end{pmatrix}=\left(\begin{array}{cccccc|cc} -L & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right)^T,$$

from which $N_m = N_c = 1$, $N_i = N_r = 0$. In this case, only the coordinate movement of joints displacing the object along the $x$-direction is possible for the system. Note that the same result is obtained if any single contact is modeled as soft-finger.

If any of the contacts are complete-constraint, all the $\mathbf{C}_{ij}$ block matrices result empty, and hence $N_m = N_c = N_i = N_r = 0$. There are no possible motions for the mechanism, as can be easily understood from direct inspection of the system.

### 4.2 Case study 2

In this example, we will consider two four-degrees-of-freedom, SCARA-type robots manipulating an object, as shown in fig. 3. Contact points are located

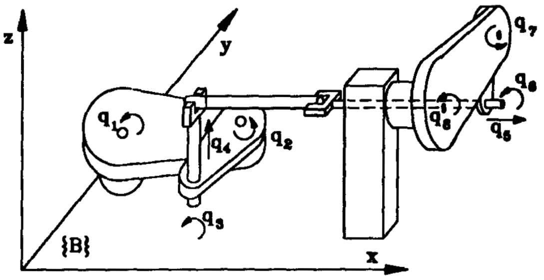


Figure 3: Two SCARA robots cooperating to manipulate an object.

at $\mathbf{c}_1 = (L\ L\ L)^T$, and $\mathbf{c}_2 = (3L\ L\ L)^T$; the corresponding normal unit vectors are $\mathbf{n}_1 = (1\ 0\ 0)^T$, and $\mathbf{n}_2 = (-1\ 0\ 0)^T$. The origin of the link reference frames are at $\mathbf{o}_1 = (0\ 2L\ 0)^T$, $\mathbf{o}_2 = (L\ 2L\ 0)^T$, $\mathbf{o}_3 = (L\ L\ 0)^T$, $\mathbf{o}_4 = (L\ L\ 0)^T$, $\mathbf{o}_5 = (4L\ L\ L)^T$ , $\mathbf{o}_6 = (4L\ L\ L)^T$, $\mathbf{o}_7 = (4L\ L\ 2L)^T$, and $\mathbf{o}_8 = (4L\ 0\ 2L)^T$, respectively. The joint axes are $\mathbf{z}_1 = \mathbf{z}_2 = \mathbf{z}_3 = \mathbf{z}_4 = (0\ 0\ 1)^T$, and $\mathbf{z}_5 = \mathbf{z}_6 = \mathbf{z}_7 = \mathbf{z}_8 = (1\ 0\ 0)^T$. Note that joints 4 and 5 are prismatic. Accordingly, the **G** and **D** matrices are:

$$\mathbf{G}=\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & -3 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & -1 & 3 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{D}^T=\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

If two complete-constraint contacts are used to model the grip on the object, we obtain

$$\mathbf{C}_{12}=\begin{pmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & L \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \qquad \mathbf{C}_{22}=\begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & L \\ L & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}.$$

The system in this configuration has connectivity $N_c = 3$, no redundancy nor indeterminacy. The feasible motions for the object are all pure translations (the last three rows of $\mathbf{C}_{12}$ are zeroes), achieved by moving the joints with suitable combinations of the columns of $\mathbf{C}_{22}$.

On the other hand, if two hard-finger contacts are assumed to be in effect (which can be imagined if the grippers in fig. 3 are substituted with two sticks), we have:

$$
\mathbf{C} = \left(\begin{array}{c|ccccc|cc}
0 & L & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & L & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & L & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\hline
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & -1 & 0 & 1 & -2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & L & -L & 0 & 0 & 0 \\
0 & L & 0 & 0 & L & -L & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & -1 & 3 & 3 & 0 & 0 \\
0 & 0 & 0 & 1 & -3 & 0 & 0 & 0
\end{array}\right).
$$

In this case, the mechanism has full connectivity ($N_c = 6$), although one of the movements of the object is quasi-statically indeterminate ($N_i = 1$). The system also exhibits redundant degrees of freedom in this configuration ($N_r = 2$). By inspection of the columns of the block matrices above, it can be easily seen that the indeterminacy is due to rotations of the object about the axis joining the two contact points, which cannot be resisted by hard-point contacts. Also, redundant motions of the arms are obtained by moving any of the third and sixth joint, i.e. rotations about the axis of the last links of the SCARA arms.

## 5 Conclusions

We discussed the mobility and kinematic analysis of robotic systems consisting of multiple arms and a common object, considering the general case in which some or all of the cooperating arms have deficient kinematic capabilities. This problem is relevant to some cases of industrial manipulation. However, the most important applications of the proposed analysis are probably in the field of dextrous manipulation. In fact, the proposed method allows to understand the structure of the instantaneous input-output relationship between joints and object velocities for the mechanism under given contact constraints: cases of multiple contacts on the same manipulator, even in passive links such as the palm, may be considered and solved. The case of single robots may be regarded as a special case of the presented technique.

Among the limitations of the proposed method, it must be noted that no information is provided as to the evolution of contact point position in case rolling contact are present. Among the activities for further development of the presented technique, the authors are addressing the problems of the static analysis (in the force domain) of cooperating manipulators, the definition of some index expressing the kinematic capabilities of the mechanisms, and the problem of the hybrid control of cooperating manipulation.


## Acknowledgements

This work has been supported partially by the CNR (Italian Research Council), under the special program on Robotics. The authors wish to thank Dr. Ken Salisbury for stimulating discussions and for providing them with a positive environment during their visit at the MIT AI Lab.


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