Allometry

Biomechanics – 16.12.19

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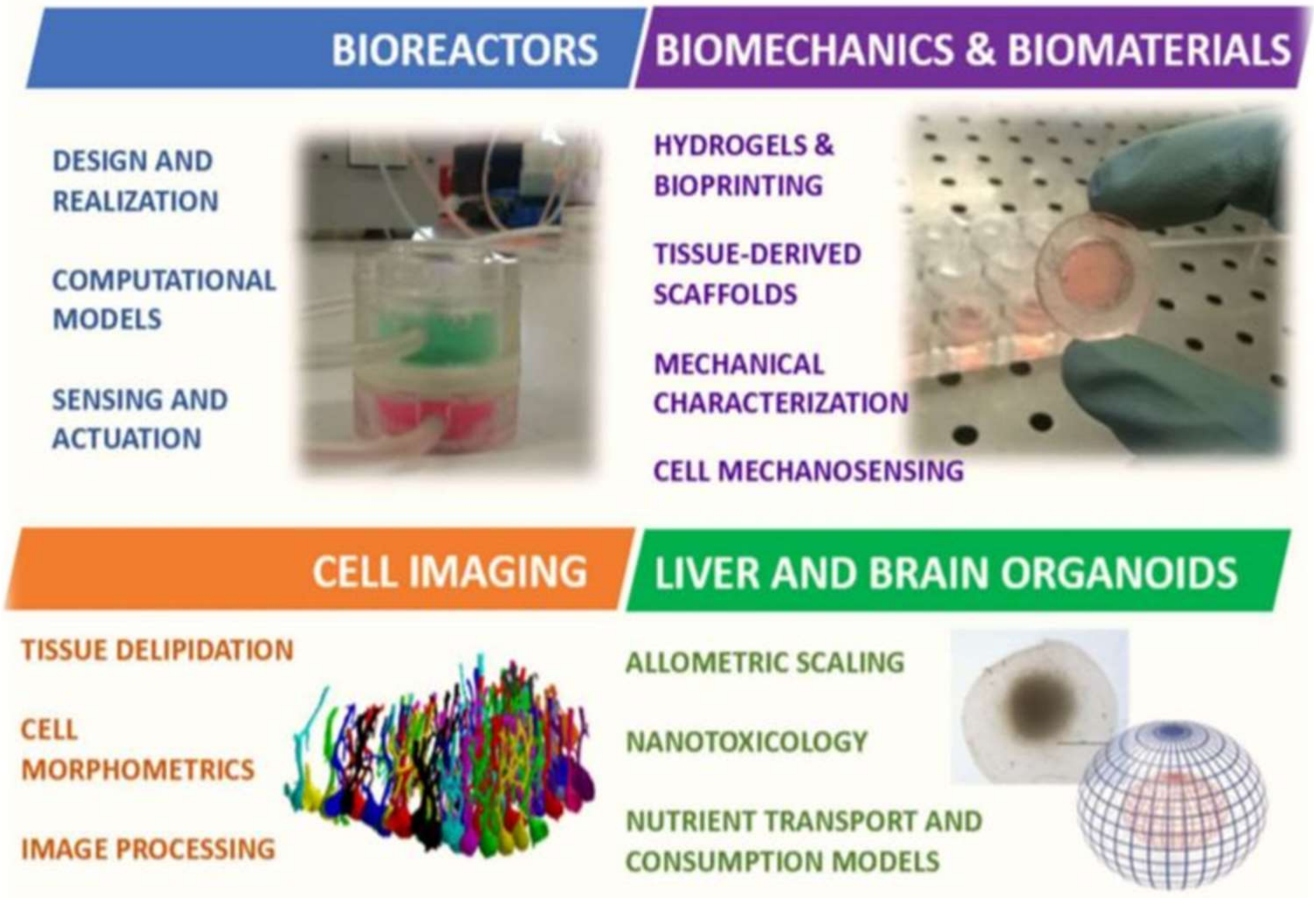

Allometry: scaling property emerging in all living organisms about characteristic physiological parameters (*e.g.* metabolic rates), which are related to body size (*i.e.* mass) through power laws.

$$\boxed{\begin{gathered} Y = aM^b \\ \log Y = \log a + b \log M \end{gathered}}$$

- *a* normalization constant (depending on *Y* and on taxonomic class)
- *b* scaling exponent (depending on *Y*)

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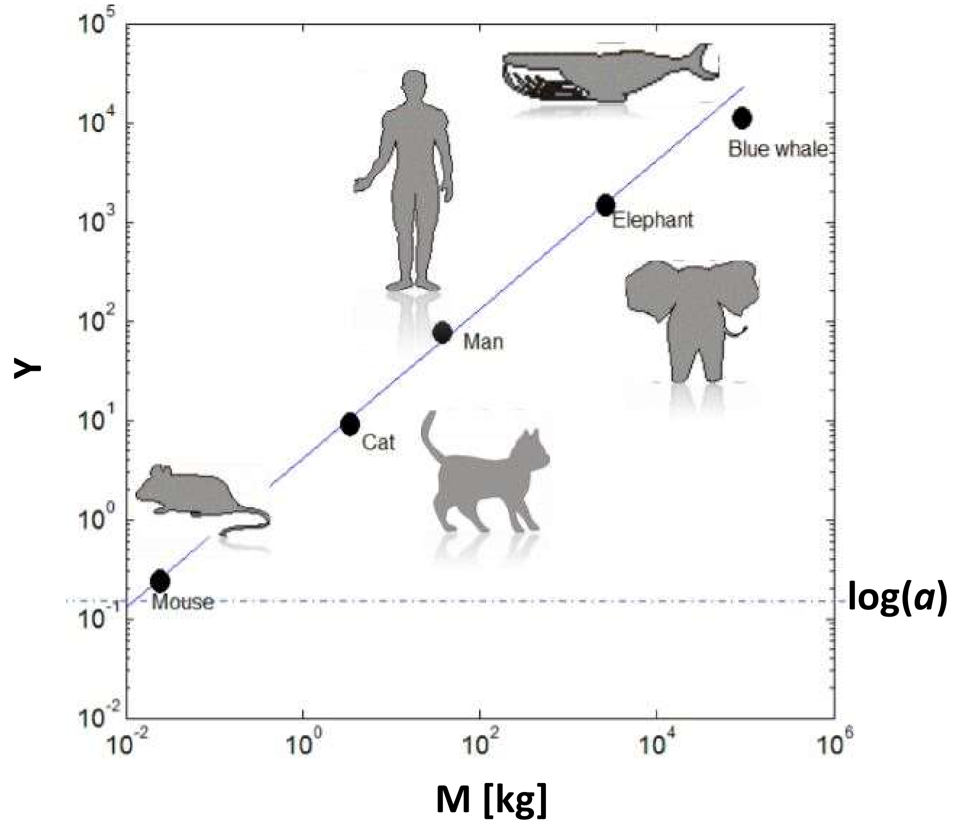

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PARAMETER	EXPONENT VALUE	MEANING
Cells size [m] Blood velocity [m/s] Pressure gradients [Pa]	b = 0	Parameter and body mass are indipendent
Volumes (bone, blood…) [m^3]	b = 1	Parameter and body mass are directly proportional (isometric scaling)
Metabolic rates [J/s] Flow rates (haematic, respiratory…) [m^3/s]	b = 3/4	Parameter increases slower than body mass
Radii of aorta and trachea [m]	b = 3/8	Parameter increases slower than body mass
Frequencies (cardiac, respiratory…) [Hz]	b = - 1/4	Parameter decreases when body mass increases
Bone mass [kg]	b = 4/3	Parameter increases faster than body mass

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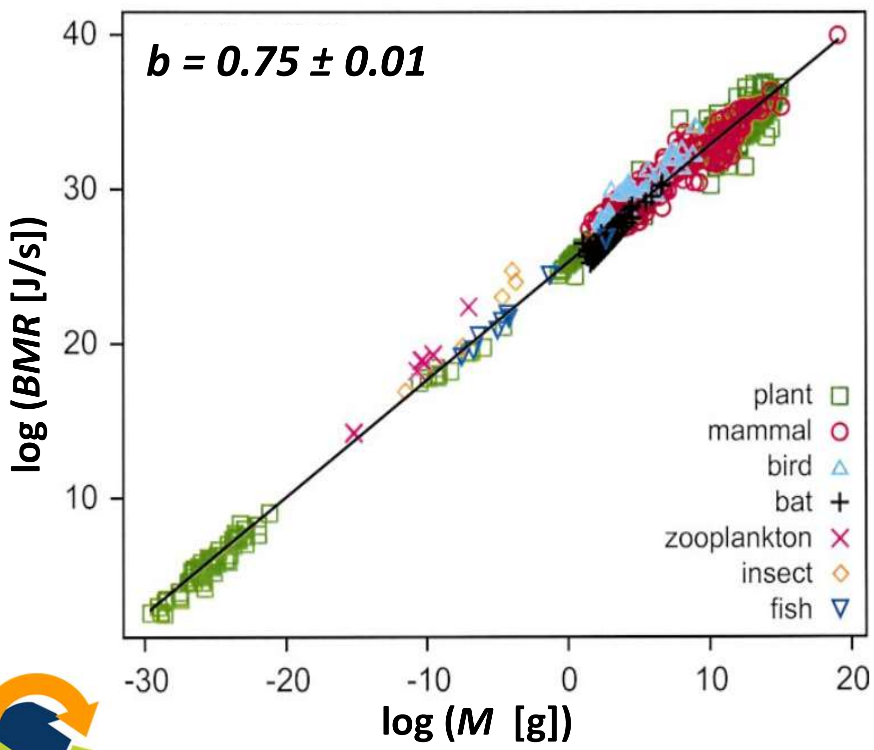
Kleiber law (KL)

$$BMR = aM^{3/4}$$



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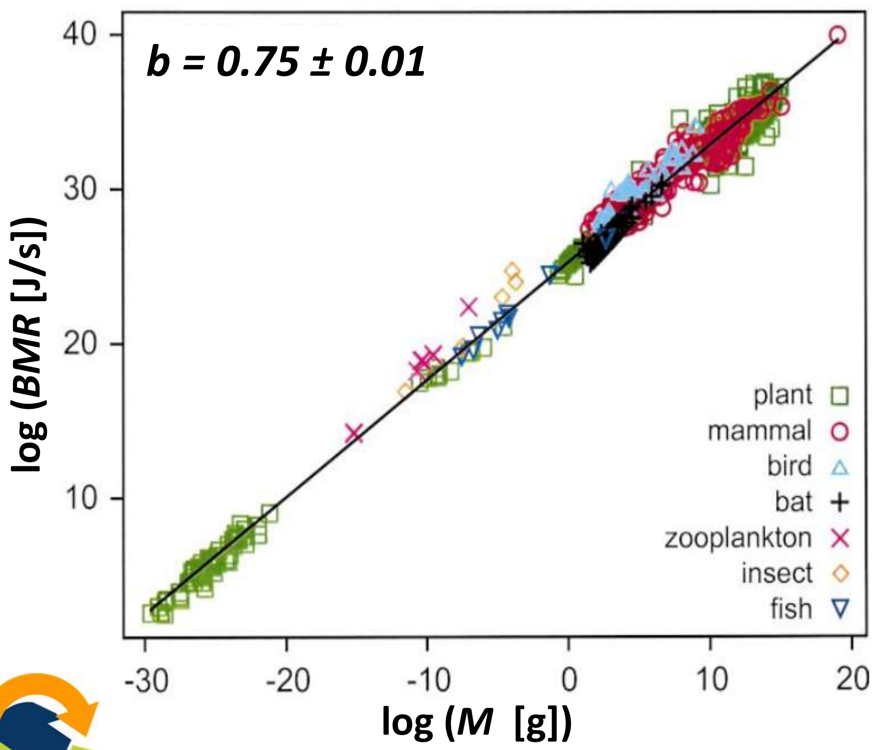
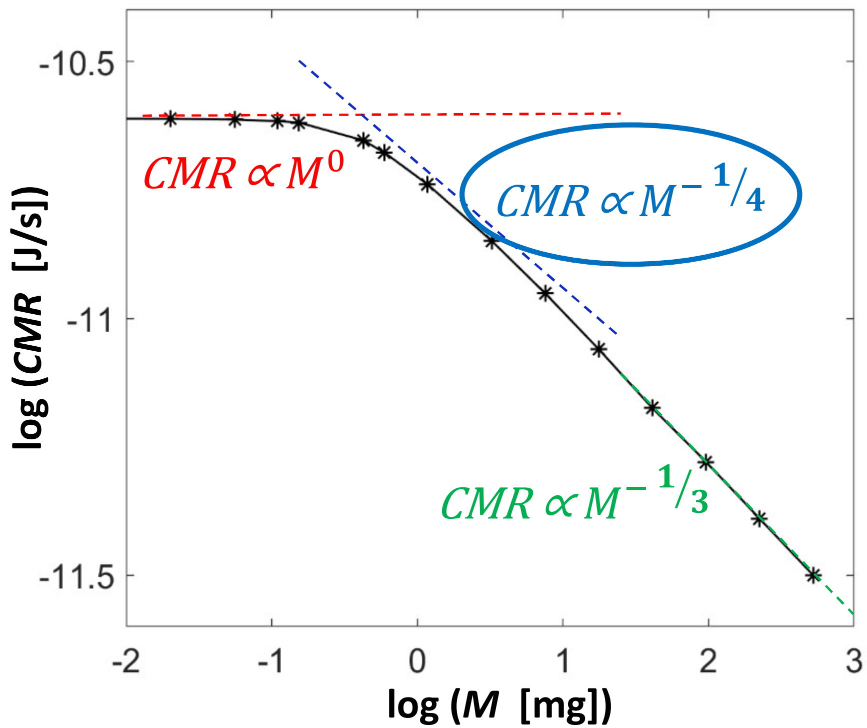


$$CMR = a'M^{-1/4}$$



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Origin of quarter-power scaling

- Geometric scaling

$$\begin{cases} A \propto l^2 \\ V \propto l^3 \\ V \propto M \end{cases} \Rightarrow \quad \boldsymbol{l \propto M^{1/3}} \qquad \boldsymbol{A \propto M^{2/3}}$$

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$$\left\{\begin{aligned} A &\propto l^2 \\ V &\propto l^3 \\ V &\propto M \end{aligned}\right. \quad \Longrightarrow \quad \begin{aligned} \boldsymbol{l} &\propto \boldsymbol{M}^{1/3} \\ \boldsymbol{A} &\propto \boldsymbol{M}^{2/3} \end{aligned}$$

Why allometric exponents are not multiples of 1/3?

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- Biological scaling
  - Stoichiometric constraints in biochemical processes
  - Integrated optimization of interdipendent sub-systems
  - Self-similar structure of nutrients supply networks

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Since **metabolic rates** (per cell, *i.e.* CMR) **underlie all physiological processes** and scale with $M^{-1/4}$, allometry is described by quarter-power scaling

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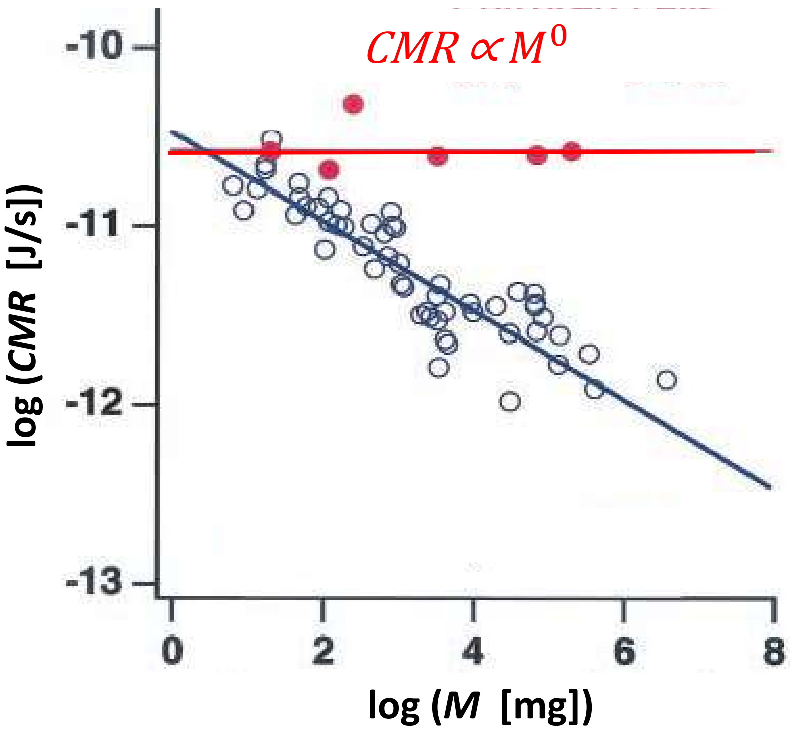
Why allometry in bioengineering?



***In vitro* monolayers**

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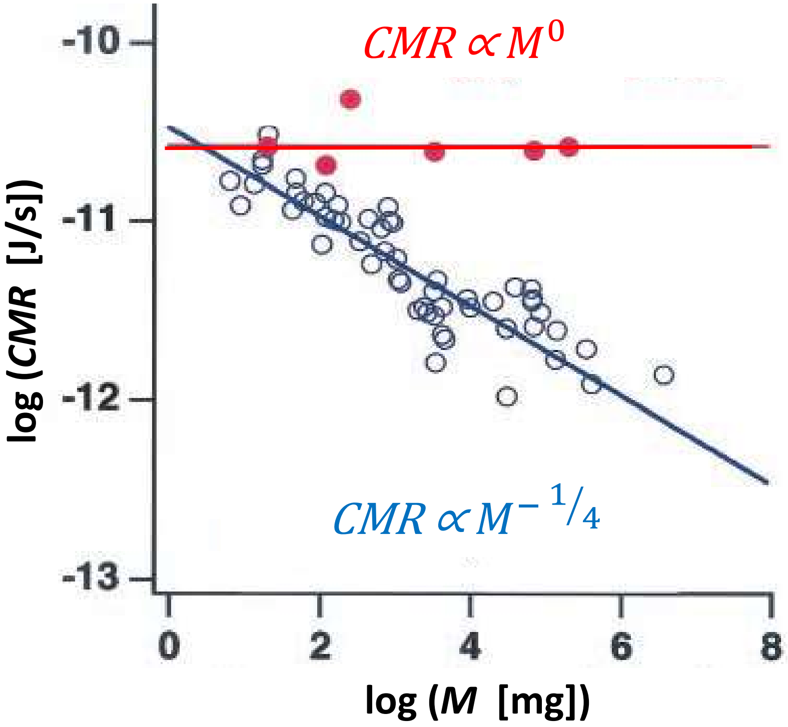
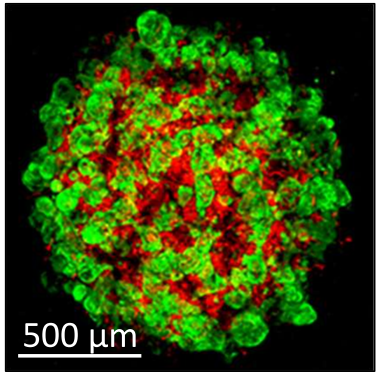


In vitro monolayers

In vivo
3D *in vitro* models



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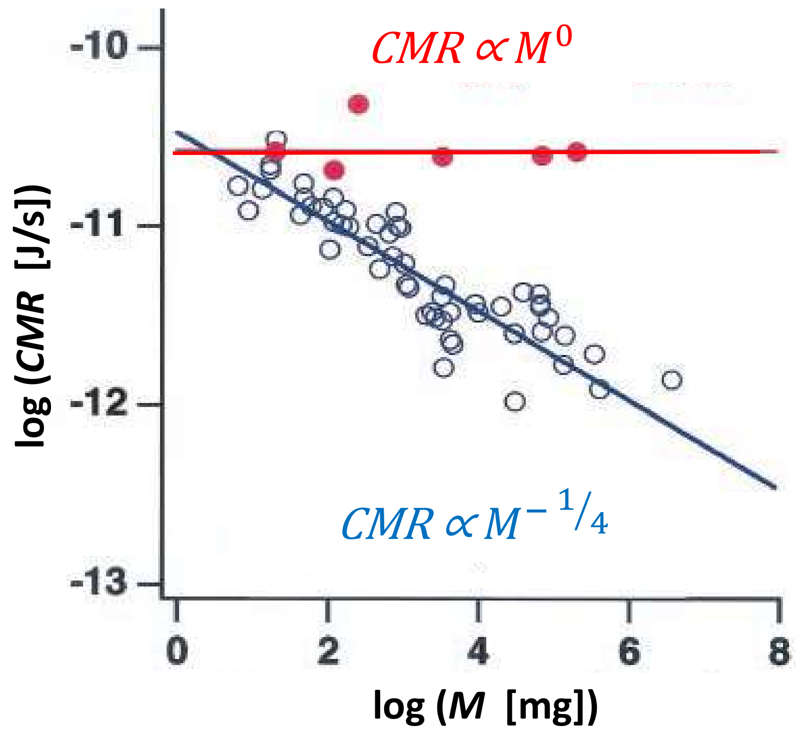

In vitro monolayers

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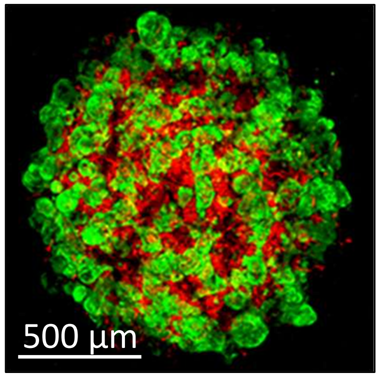

Since **KL** is a **universal law for biological systems**, we need to account for it to design predictive and **physiologically relevant *in vitro* models**!

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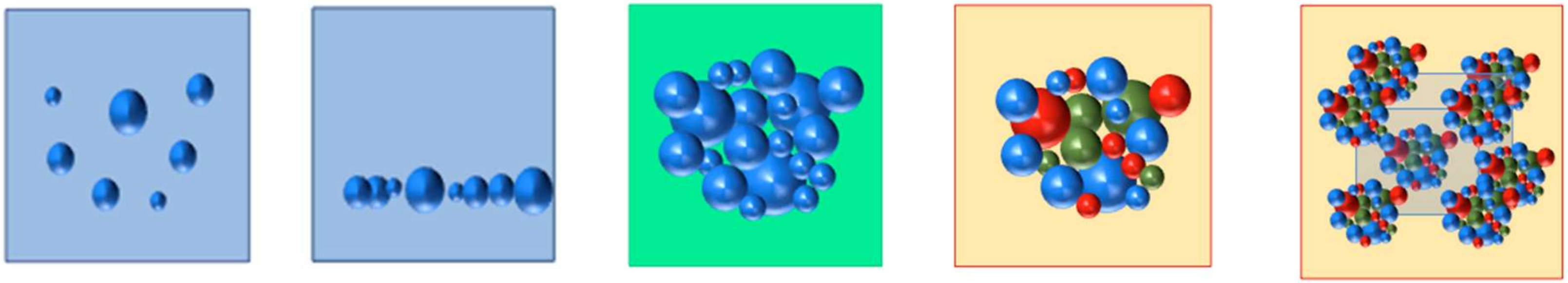

INCREASING COMPLEXITY

Which size level does allometric scaling start from?

Which is the size range allowing allometric scaling to emerge?

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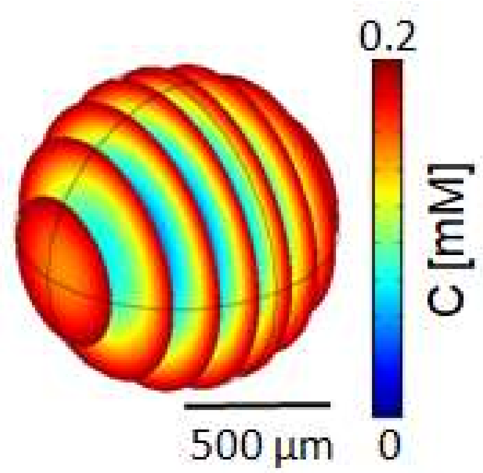
Case of study: liver and brain *in vitro* models

In silico **modelling: diffusion and consumption**

$$\frac{\partial c_{O_2}}{\partial t} = \nabla^2 \left(D_{O_2} c_{O_2} \right) + R = \nabla^2 \left(D_{O_2} c_{O_2} \right) - \frac{V_{max} c_{O_2}}{k_M + c_{O_2}}$$



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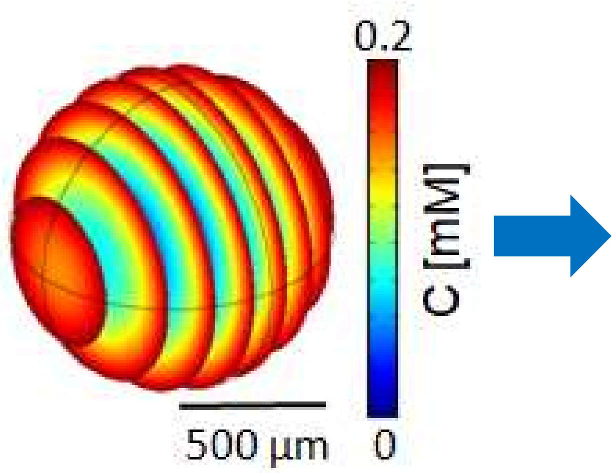

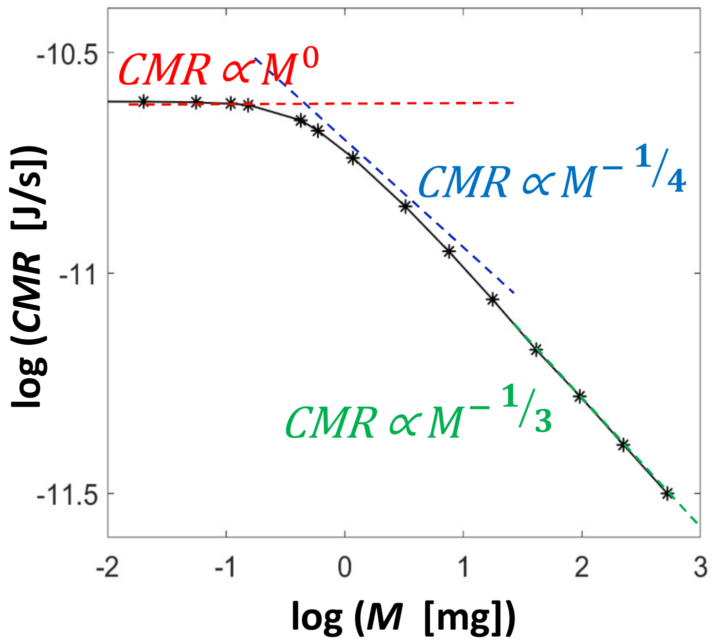

$$B_{O_2} = -\Delta H \oiint \underline{J}_{O_2} \cdot \partial \underline{A} = -\Delta H \oiint \underline{\nabla}\left(D_{O_2} c_{O_2}\right) \cdot \partial \underline{A}$$

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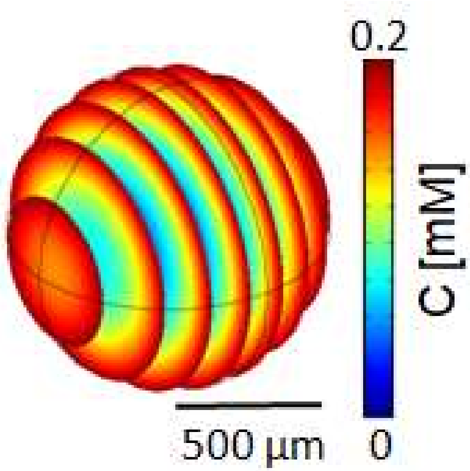

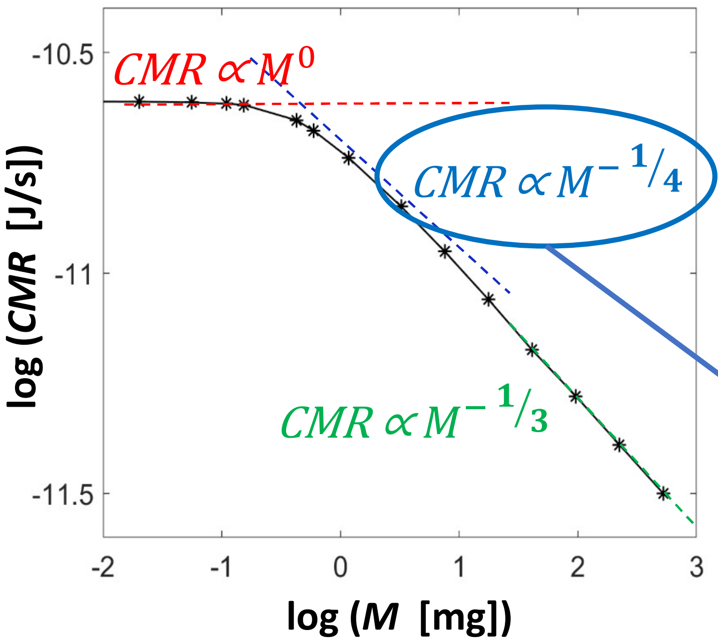

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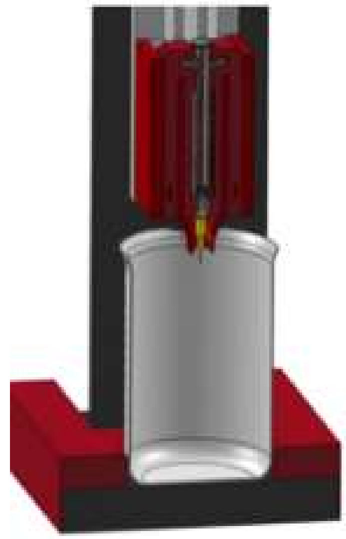
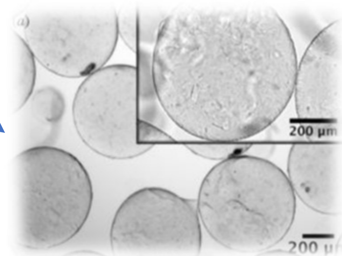
Fabrication, *in vitro* **O_2 measurements** and **validation**





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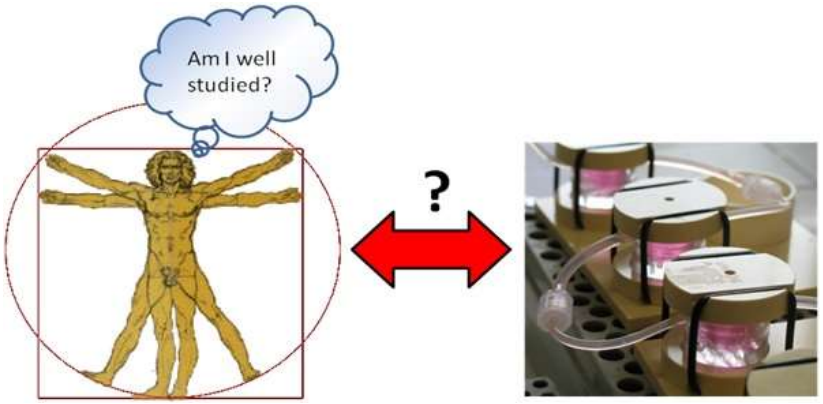
Thesis are available on these topics!



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