Allometric scaling

Biomechanics - 14.12.20

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BIOREACTORS

DESIGN AND REALIZATION

COMPUTATIONAL MODELS

SENSING AND ACTUATION

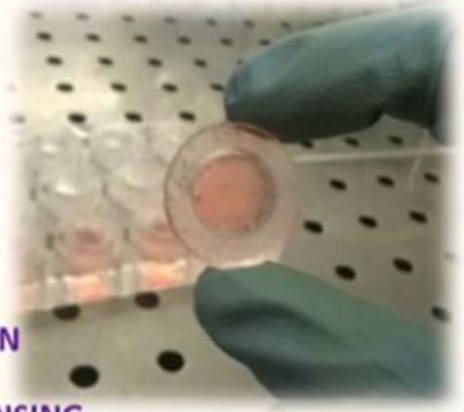
BIOMECHANICS & BIOMATERIALS

HYDROGELS & BIOPRINTING

TISSUE-DERIVED SCAFFOLDS

MECHANICAL CHARACTERIZATION

CELL MECHANOSENSING

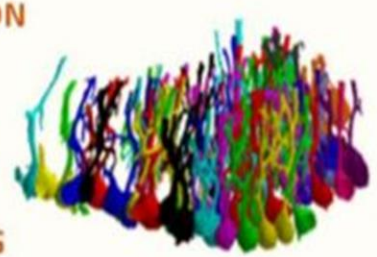


CELL IMAGING

TISSUE DELIPIDATION

CELL MORPHOMETRICS

IMAGE PROCESSING

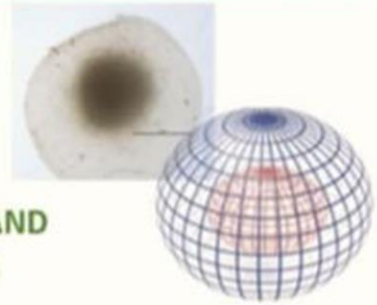


LIVER AND BRAIN ORGANOIDS

ALLOMETRIC SCALING

NANOTOXICOLOGY

NUTRIENT TRANSPORT AND CONSUMPTION MODELS



Allometric scaling: intrinsic feature of all living organisms about characteristic physiological parameters (*e.g.* metabolic rates), which are related to body size (*i.e.* mass) through power laws.

$$Y = aM^b$$

$$\log Y = \log a + b \log M$$

- a normalization constant (depending on Y and on taxonomic class)
- b scaling exponent (depending on Y)

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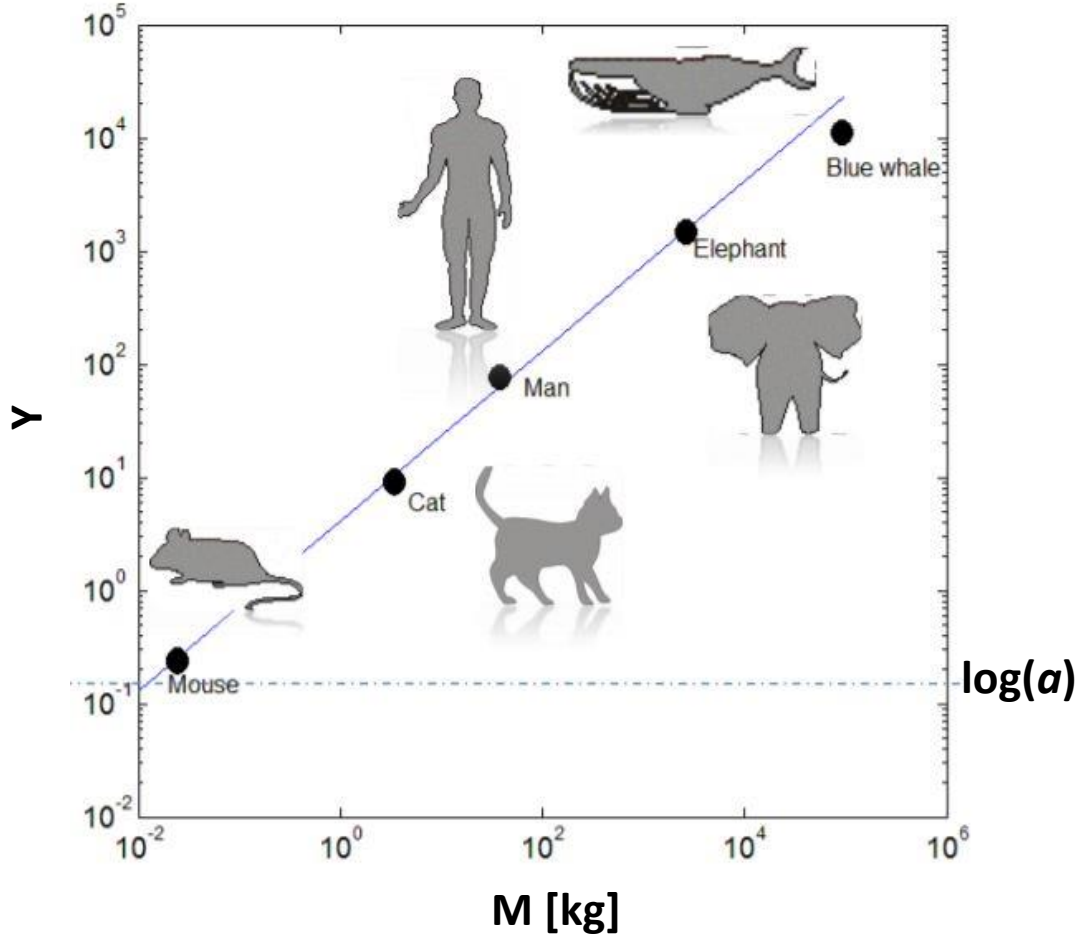

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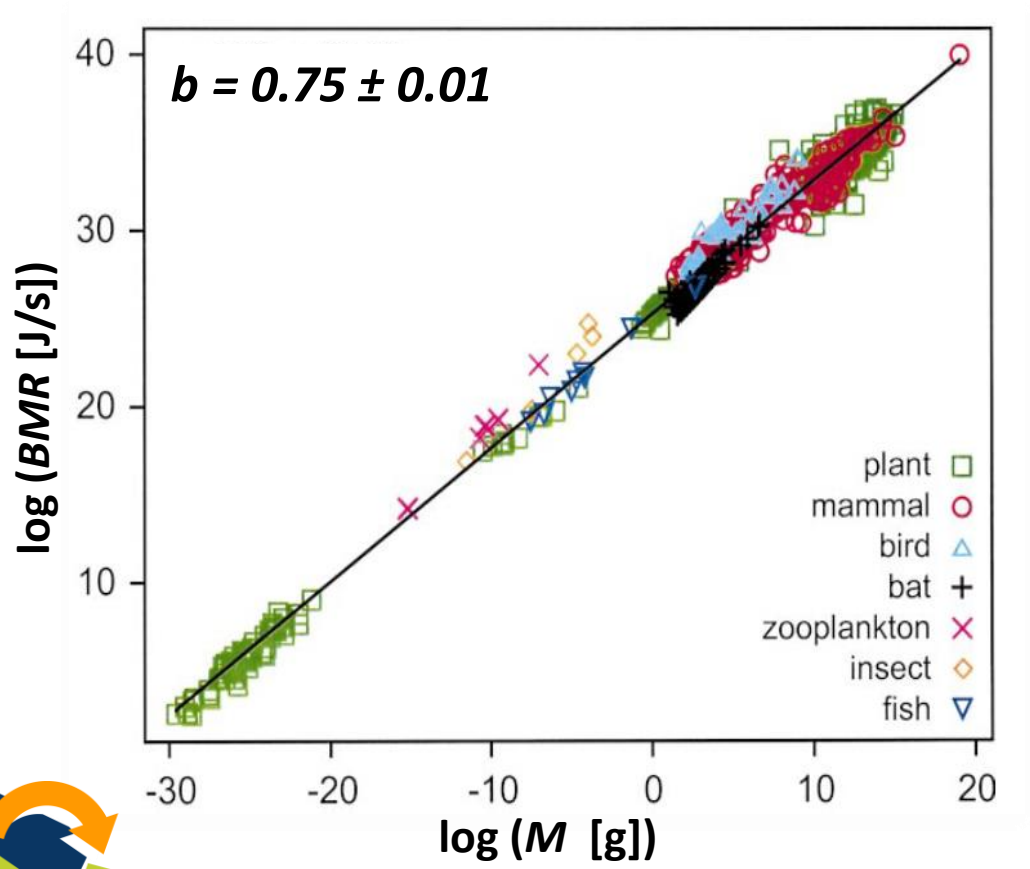
PARAMETER	EXPONENT VALUE	MEANING
Cells size [m] Blood velocity [m/s] Pressure gradients [Pa]	b = 0	Parameter and body mass are indipendent
Volumes (bone, blood…) [m^3]	b = 1	Parameter and body mass are directly proportional (isometric scaling)
Metabolic rates [J/s] Flow rates (haematic, respiratory…) [m^3/s]	b = 3/4	Parameter increases slower than body mass
Radii of aorta and trachea [m]	b = 3/8	Parameter increases slower than body mass
Frequencies (cardiac, respiratory…) [Hz]	b = - 1/4	Parameter decreases when body mass increases
Bone mass [kg]	b = 4/3	Parameter increases faster than body mass


NAT GEO
WILD

Kleiber's law (KL)

$$BMR = aM^{3/4}$$



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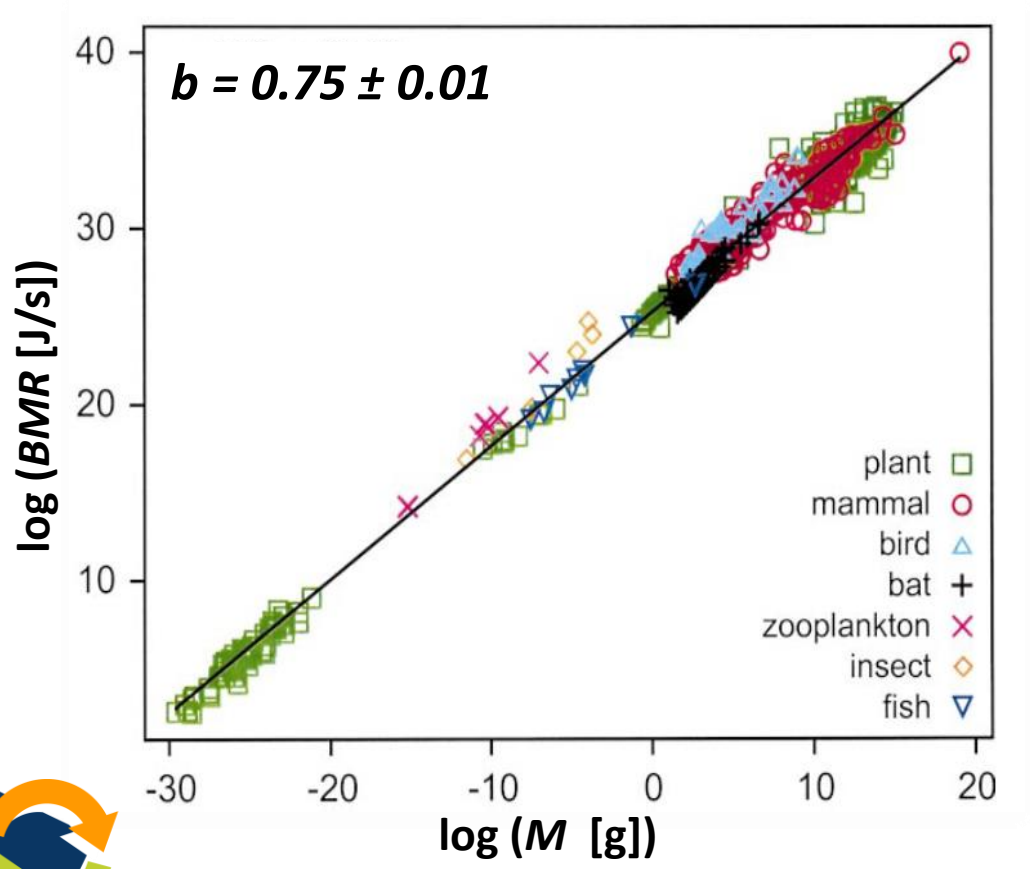
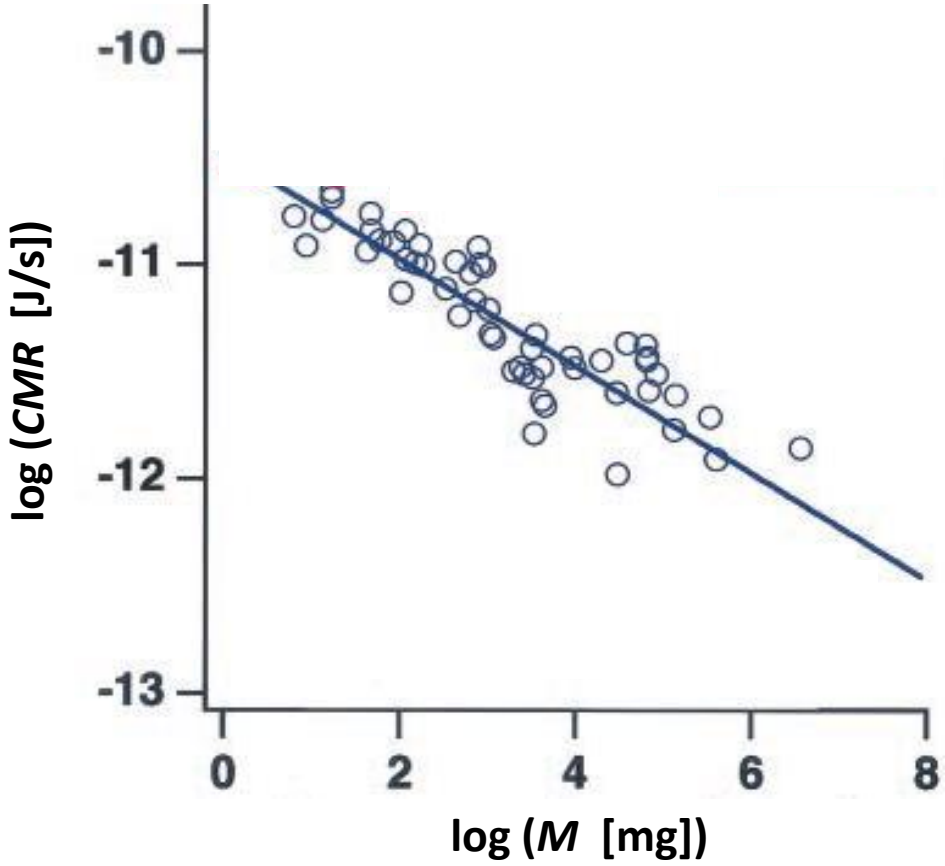
Kleiber's law (KL)

$$BMR = aM^{3/4}$$

$$CMR = a'M^{-1/4}$$





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Origin of quarter-power scaling

- Geometric scaling

$$A \propto l^2$$
$$V \propto l^3$$
$$V \propto M$$

$$\boldsymbol{l \propto M^{1/3}}$$
$$\boldsymbol{A \propto M^{2/3}}$$

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Origin of quarter-power scaling

- Geometric scaling

$$\begin{cases} A \propto l^2 \\ V \propto l^3 \\ V \propto M \end{cases} \Rightarrow \begin{cases} \boldsymbol{l \propto M^{1/3}} \\ \boldsymbol{A \propto M^{2/3}} \end{cases}$$

Why allometric exponents are not multiples of 1/3?

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- Biological scaling
  - Stoichiometric constraints in biochemical processes
  - Integrated optimization of interdipendent sub-systems
  - Self-similar structure of nutrients supply networks

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Since **metabolism underpins** the great majority of **physiological processes** and scales with $M^{3/4}$, allometry is described by quarter-power scaling

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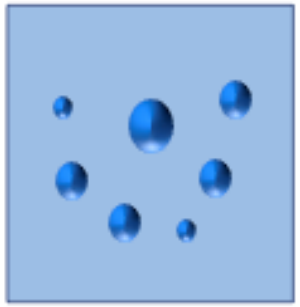
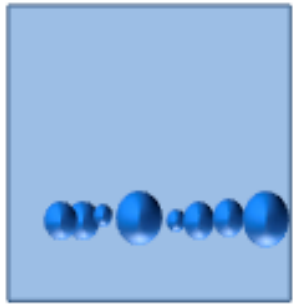
Why allometric scaling in bioengineering?

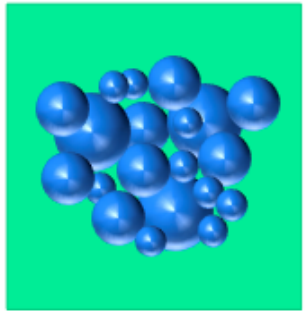
Single cells



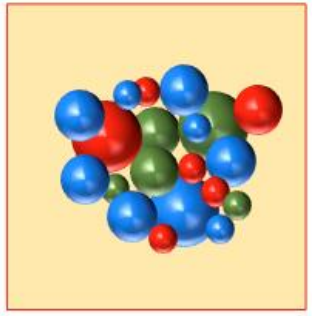
2D monolayers



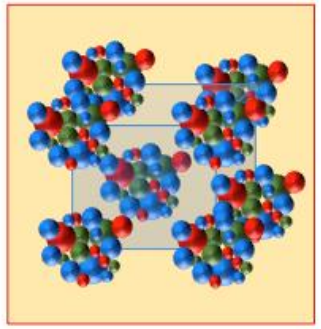
Spheroids



Organoids



3D architectures of spheroids/organoids



INCREASING COMPLEXITY

Which size level does allometric scaling start from?

Which is the size range allowing allometric scaling to emerge?

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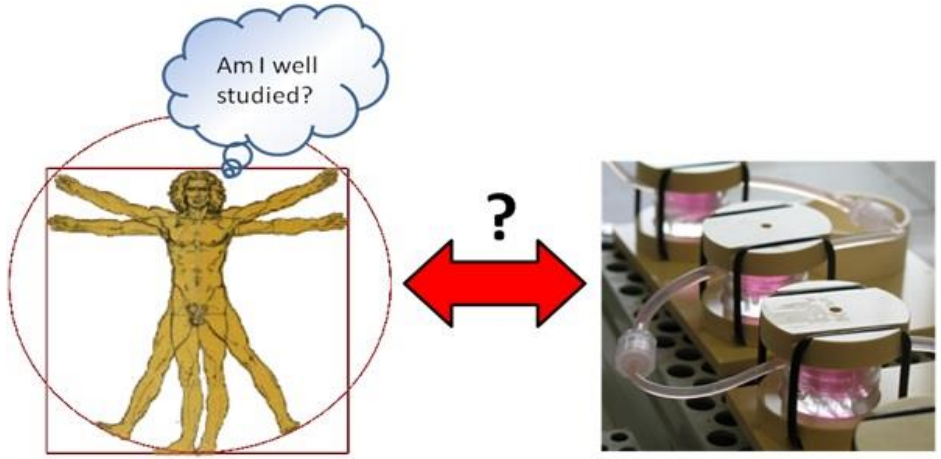
Thesis are available on these topics!



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