Mobile robot algebraic localizability

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2. Setup and assumptions for localization of unicycle
3. Localizability
4. Simulation and experimental result
5. Conclusion

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Challenge for autonomous navigation: "Where am I?"

☞ real-time localization problem:

Real time estimation of the robot's pose
(or posture = position + orientation)
using noisy measurements

Introduction: Absolute positioning with Landmarks

☞ landmarks = features with known coordinates (relative measure).

☞ "human made" can be passive or active (ex. radio signal).

- *active landmarks* as in [14] (wifi infrastructure) and [15] (bluetooth protocol): lack of accuracy.
- [16] uses magnetic patterns (active landmarks): needs environment modifications to work (drawback).

☞ Avoid to modify the environment ⇒ natural landmarks: laser scanners, ultrasound sensors (distance) or cameras (distance, angle).

Introduction and related works

☞ Trends for AGV: move to optically guided systems since the above mentioned solutions (GPS, active landmarks and wire-guided solutions) have important drawbacks.

☞ Our goal: design and analyze landmark based solution for a unicycle mobile robot equipped with a monocular camera (with **one** or two landmarks).

☞ Monocular camera (extract natural landmarks): corner points ([17, 18]) + other algorithms SIFT [19] or SURF [20].

☞ Single landmark based solution can be used also in the multi-landmarks case by fusing the data in order to improve the localization algorithm.

Introduction and related works

☞ Step 1: Vision sensors ⇒ information,

☞ Localization problem ∼ observation problem [26]:

- Step 2: localizability is related to the observability problem (see [27, 28, 29]).
- Step 3 observer/estimator design.

☞ Here in our algebraic approach Step 2 and 3 and merged.

Introduction and related works

Step 2:Localizability/observability:

- Linearized approximations: NO ([27]),
- NL observability Hermann and Krener [30, 31]: YES ([28]).

Step 3: Observer design [26]:

- [28]: nonlinear Luenberger-like observer combined with the projection of stationary landmarks,
- [27]: extended Kalman filter (EKF) in leader-follower context and observability can be tested through the Extended Output Jacobian matrix.
- [28, 29] (extension to the SLAM problem): CNS (observable) = two landmarks, estimation error is minimized using some optimal control methods/

Introduction and related works

☞ EKF cons.: needs the inputs knowledge + noise,

☞ Our solution based on differential algebraic setting + efficient numerical derivation of noisy signals = good real-time localization (estimation of pose and velocities) using bearing only measurements.

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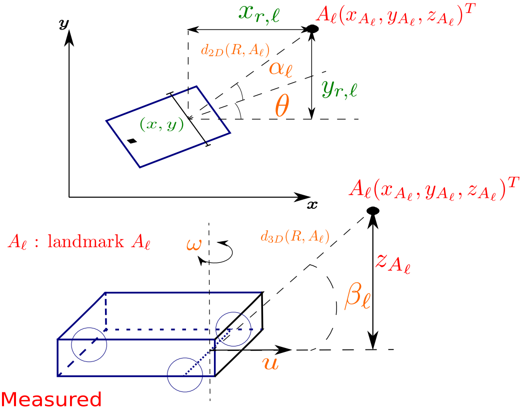
Setup and assumptions for localization of unicycle

Localization = pose estimation w.r.t fixed frame.
Setup:

- mobile robot with its kinematic model,
- sensors with its measurement model

The robot - An image of the camera



Measured
Depends of sensors assumption
Estimated

Robot and landmark notation for the localization

Setup and assumptions for localization of unicycle

☞ POI + image processing = angles:

$$\alpha_\ell = \arctan\left(\frac{y_{r,\ell}}{x_{r,\ell}}\right) - \theta, \tag{1}$$

$$\beta_\ell = \arctan\left(\frac{z_{A_\ell}}{\sqrt{(x_{r,\ell})^2 + (y_{r,\ell})^2}}\right). \tag{2}$$

Measured output:

$$\mathbf{y}_m = \mathbf{y} + \varpi, \tag{3}$$

Setup and assumptions for localization of unicycle

Any other practical sensor setup could be consider as soon as we arrive at one of the following fives cases given in table:

Measurement assumptions	Number of Landmarks	Measures	Output (3)
MA1	1	azimuth angle elevation angle	$\mathbf{y} = \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix}$
MA2	1 + Compass	azimuth angle elevation angle orientation	$\mathbf{y} = \begin{pmatrix} \alpha_1 \\ \beta_1 \\ \theta \end{pmatrix}$
MA3	2	2 elevation angles	$\mathbf{y} = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$
MA4	2	2 azimuth angles	$\mathbf{y} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$
MA5	2	azimuth angle of A_1 elevation angle of A_2	$\mathbf{y} = \begin{pmatrix} \alpha_1 \\ \beta_2 \end{pmatrix}$

Measurement assumptions: α_ℓ and β_ℓ are given by (1), (2) together Notations.

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   - Observability vs. Localizability
   - Algebraic localizability
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☞ Unicycle robot: localizability = observability.

☞ Car (see [8, 38]), the state vector is $(x, y, \theta, \varphi)^T$ and its kinematic model is:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \end{pmatrix} = \begin{pmatrix} L\cos(\theta)\sin(\varphi) & 0 \\ -L\sin(\theta)\sin(\varphi) & 0 \\ \cos(\varphi) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}. \qquad (4)$$

pose is a sub-vector of the state vector.

☞ observability ⇒ localizability (reverse is false).

Localizability: definition and remarks

☞ Similarity between localizability/observability was already mentioned in [27, 28, 29].

☞ linearized approximations can be non observable [27], thus localization is not possible using such approximations.

☞ differential nonlinear systems theory proves the feasibility to reconstruct the state, thus the localization problem is solvable (see [28] and [29]).

☞ Usually, real-time localization solutions relies on non linear observer design.

Observability review: differential geometric framework

Hermann and Krener in [30] obtained sufficient conditions *local observability rank condition* from which we can deduce:

Theorem

A unicycle type mobile robot is **localizable** *at a point* $\mathbf{x}_0$ *if the co-distribution of observability is of the form* $d\mathcal{O}_{\mathbf{x}_0}(\mathbf{h}) = \mathcal{O}(\mathbf{x}_0)(dx, dy, d\theta)^T$ *with* $rank(\mathcal{O}(\mathbf{x}_0)) = 3$.

Measures: $\mathbf{y} = \mathbf{h}(\mathbf{x})$
State: $\mathbf{x}$. Here for (**??**), we have $\mathbf{x} = \mathbf{p}$.
Pose: $\mathbf{p} = (x, y, \theta)$.

Observability review: differential geometric framework

Example (Trivial case: the position is measured)

☞ Measured output: $\mathbf{y} = \mathbf{h}(\mathbf{x}) = (x, y)^T$.

☞ $d\mathcal{O}_{\mathbf{x}_0}(\mathbf{h}) = \mathrm{span}\{dx, dy, d(u\cos(\theta)), d(u\sin(\theta))\} = \mathrm{span}\{dx, dy, d\theta\}$.

☞ $\dim d\mathcal{O}_{\mathbf{x}_0}(\mathbf{h}) = 3 \Rightarrow$ localizable.

Observability review: differential geometric framework

Example (Case 1: the bearing angles are measured for one landmark (MA1))

☞ One landmark A_1 + measurement $\mathbf{h}(\mathbf{p}) = (\alpha_1, \beta_1)^T$,

☞ There is no closed form for $dL_{\mathbf{f}}^{\ell-1}\mathbf{h}(\mathbf{x})$.

☞ Lengthly computation of $d\mathcal{O}_{\mathbf{x}_0}(\mathbf{h})$ prove that the rank is 2: the unicycle mobile robot is not localizable with only one landmark.

Observability review: differential algebraic framework

☞ local observability rank condition is concerned with the successive output time derivatives.

☞ For an introduction to Diff. Alegbra (for more details see [39, 40, 41, 34, 42, 43]).

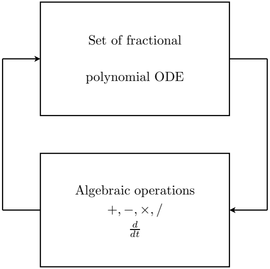
Observability review: differential algebraic framework

Some hints about the algebraic concepts and tools:

- Ring (or Field) $\rightarrow$ differential Ring (Field) (as soon as derivation is defined Leibniz rule),
- differential ideal,
- differential extension $(\mathbf{L}/\mathbf{F}$: one is bigger than the other $\mathbf{F} \subset \mathbf{L})$,
- notion of basis (generator and freeness), dimension of the basis is the differential transcendence degree,
- etc ...

I will not give details but will give a down to earth vision.

Observability review: differential algebraic framework



☞ suitable for all real processes: $\exp, \log$ trigonometric functions and so on satisfy ODE.

$$x_1\ddot{x}_1 + x_1^2\dot{x}_1 + x_1^2 + \exp(x_1) = 0 \quad \underset{x_2 = \exp(x_1)}{\Leftrightarrow} \quad \begin{array}{c} x_1\ddot{x}_1 + x_1^2\dot{x}_1 + x_1^2 + x_2 = 0, \\ \dot{x}_2 - x_2\dot{x}_1 = 0. \end{array}$$

$$\dot{\mathbf{p}} = \mathbf{f}(\mathbf{p}, \mathbf{u}) = \begin{pmatrix} v\cos(\theta) \\ v\sin(\theta) \\ \omega \end{pmatrix}, \tag{5}$$

Fit this framework (trigonometric functions) using:

$$\begin{cases} z &= x + iy \\ \Theta &= \exp(i\theta) \end{cases}, \tag{6}$$

(5) can be rewritten as:

$$\dot{z} = \Theta v, \tag{7}$$

$$\dot{\Theta} = i\Theta\omega. \tag{8}$$

Observability review: differential algebraic framework

Set of all differential algebraic relations using variables $W_1, \ldots, W_n$

System ODE $(P_\ell)_{\ell=1}^r$

$\mathbf{I}\{P_1, \ldots, P_r\}$

ideal (prime ?)

If YES $\Rightarrow$ no zero divisor

fractional field
$\mathbf{F} :=$
$\mathbf{k}\{W_1, \ldots, W_n\}/\mathbf{I}\{P_1, \ldots, P_r\}$

Non Linear Sys.

$\mathbf{F}/\mathbf{k}$

Properties: invertibility, flatness, ...

Simple characterization of the intrinsic properties of a system: invertibility, flatness (closed to controllability) [45, 46, 47, 48, 38], ...

Observability: (see [39] and [40])

Definition

$\zeta \in \mathbf{F}$ is observable w.r.t. $\mathbf{z} := \{z_\ell : \ell \in I\} \subset \mathbf{F}$ if it is algebraic over $\mathbf{k}\langle \mathbf{z} \rangle$.

☞ $\zeta =$ algebraic function of the components of $\mathbf{z}$ and a finite number of their derivatives.

Theorem

A non linear system is observable if, and only if, any state variable is a function of the input and the output variables and their derivatives up to some finite order.

$$
\begin{aligned}
\dot{x}_1 &= x_2 + f_1(x_1), \\
\dot{x}_2 &= f_2(x_1, x_2), \\
y &= x_1
\end{aligned}
$$

Algebraic localizability

Theorem

A mobile robot is **localizable** *with respect to* $\mathbf{u}, \mathbf{y}$ *if, and only if, there is a non-zero irreducible differential polynomial linking the pose to the measured output* $\mathbf{y}$*, the input* $\mathbf{u}$ *and a finite number of their time derivatives.*

Example (Position is measured)

$$\dot{z} = \Theta v, \tag{9}$$

$$\dot{\Theta} = i\Theta\omega. \tag{10}$$

Output $\mathbf{y} = z = x + iy$ (two sensors).

Observable: $\mathbf{p_a} = \left(\mathbf{y}, \Theta = \frac{\dot{\mathbf{y}}}{v}\right)^T$.

Algebraic localizability

Example (MA1: bearing angles for one landmark)

$$\dot{\mathbf{y}} = -v - i\omega\mathbf{y}. \tag{11}$$

Θ cannot be obtained as an algebraic relation in terms of $\mathbf{y}, v, \omega$ and a finite number of their derivatives: **robot is not localizable**.

☞ localizability defect is defined using an algebraic setting and corresponds to the number of variables within the list z, Θ to be added to $\mathbf{u}, \mathbf{y}$ in order to retrieve the robot localizability.

- ☛ All cases MA2 to MA4 are solved (similar technics).

- ☛ Is there anything in between MA1 and MA2 ?

MA1-BIS: azimuth and elevation angles are measured for one landmark and one target: 4 outputs $y_1, \ldots, y_4$

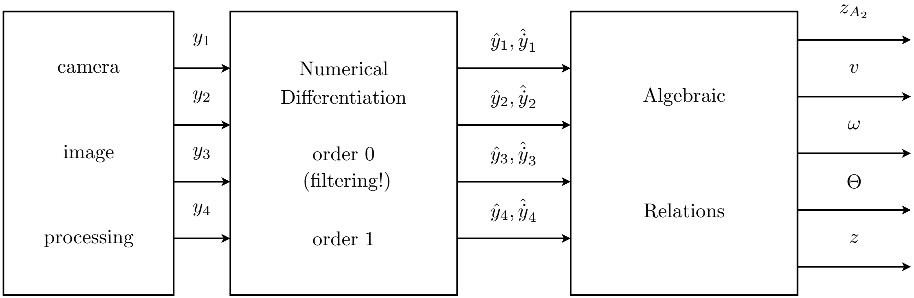
$$v = \frac{2 z_{A_1} \dot{y}_2 y_1}{(1+y_1^2) y_2^2} \tag{12}$$

$$z_{A_2} = \frac{z_{A_1} \dot{y}_2 y_1}{(1+y_1^2) y_2^2} \times \frac{(1+y_3^2) y_4^2}{\dot{y}_4 y_3} \tag{13}$$

$$\omega = -i \frac{z_{A_1} y_4^2(t) \left(y_1(t)\dot{y}_2(t) - \dot{y}_1(t) y_2(t)\right) + z_{A_2} y_2^2(t) \left(\dot{y}_3(t) y_4(t) - y_3(t)\dot{y}_4(t)\right)}{z_{A_1} y_1(t) y_2(t) y_4^2(t) - z_{A_2} y_2^2(t) y_3(t) y_4(t)} \tag{14}$$

$$\Theta = \Theta(t_0) \frac{z_{A_1} \frac{y_1(t_0)}{y_2(t_0)} - z_{A_2} \frac{y_3(t_0)}{y_4(t_0)}}{z_{A_1} \frac{y_1(t)}{y_2(t)} - z_{A_2} \frac{y_3(t)}{y_4(t)}} \tag{15}$$

$$z(t) = c_{A_1} - z_{A_1} \Theta \frac{y_1(t)}{y_2(t)} \tag{16}$$



Need good numerical differentiation in noisy environment

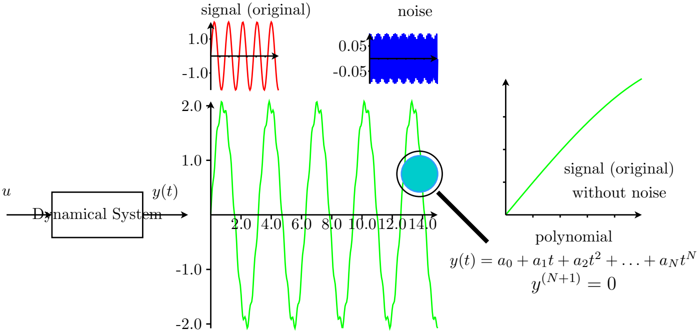
Algebraic localizability

Algorithms:	Non-A Algorithm	EKF
Input (known):	NO	YES
Noise characteristics (known)	NO	YES
θ measured	NO	NO
Initialization	NO	YES
Only one landmarks	YES	NO
Confidence interval	NO (TBD)	YES

Table: Algorithms Comparison

Localization as a numerical differentiation problem

All we want to know is in the output signal...



Real-time estimations of derivatives for noisy signals

Takes $x^N = 0$ (Laplace) + alg. annihilator $\Rightarrow$ time domain

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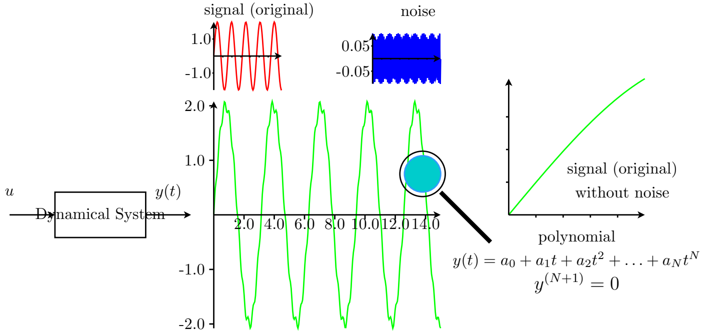

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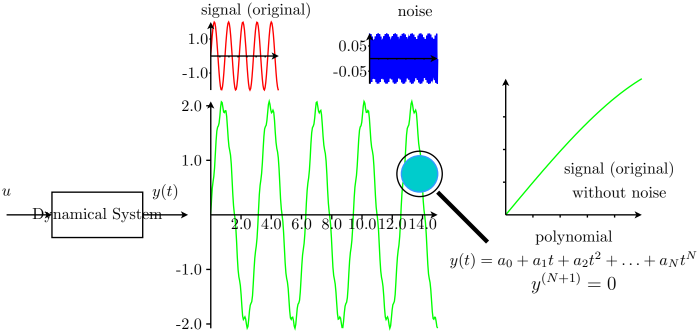

Real-time estimations of derivatives for noisy signals

Takes $x^N = 0$ (Laplace) + alg. annihilator $\Rightarrow$ time domain

☞ Estimate $x^{(2)}(0)$ through the truncated series of order 2:

$$\mathcal{R}: \qquad X = \frac{x(0)}{s} + \frac{x^{(1)}(0)}{s^2} + \frac{x^{(2)}(0)}{s^3}$$

Idea: kill undesired terms (blue) except the one to estimate (red)

Step 1 $\times s^2$: $s^2 X = s x(0) + x^{(1)}(0) + \frac{x^{(2)}(0)}{s}$

Step 2 $\frac{d^2}{ds^2}$: $2X + 4s\frac{dX}{ds} + s^2\frac{d^2X}{ds^2} = \frac{2}{s^3}x^{(2)}(0)$

Step 3 $\times \frac{1}{s^3}$: $\frac{2}{s^3}X + \frac{4}{s^2}\frac{dX}{ds} + \frac{1}{s}\frac{d^2X}{ds^2} = \frac{2}{s^6}x^{(2)}(0)$

Step 4 Go back to the time domain:

$$\frac{2T^5}{5!}x^{(2)}(0) = T^3\int_0^1 \left(2w^{2,0}(\tau) - 4w^{1,1}(\tau) + w^{0,2}(\tau)\right) y(T\tau)d\tau$$

Localization as a numerical differentiation problem

Numerical derivation

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$$2X + 4s\frac{dX}{ds} + s^2\frac{d^2X}{ds^2} = \frac{2}{s^3}x^{(2)}(0)$$

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Non-A INRIA
LAGIS

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Localization as a numerical differentiation problem

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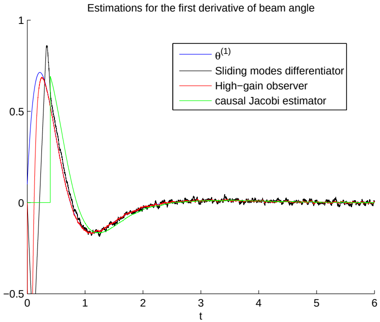
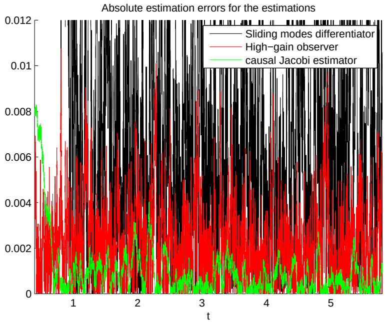
The Ball and Beam system:





$SNR = 24.5\text{dB}$ and $T_s = 10^{-4}$

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Simulations: EKF knows the noise characteristic

Algorithm	Results			
	distance error mean	distance error variance	v error mean	ω error mean
NON-A-MA1BIS	0.2196	0.0040	0.0334	0.0015
NON-A-MA2	0.1422	0.0034	0.0286	3.2435e-006
NON-A-MA3	0.1869	0.0033	0.5000	0.0100
EKF-MA3	0.2219	0.0122	X	X
NON-A-MA4	1.6806	1.8969	X	X
NON-A-MA5	1.3127	1.4534	X	X

Simulations: EKF does not know the noise characteristic

Algorithm	Results			
	distance error mean	distance error variance	v error mean	ω error mean
NON-A-MA1BIS	0.2081	0.0039	0.0334	0.0015
NON-A-MA2	0.1436	0.0035	0.0287	3.2435e-006
NON-A-MA3	0.1839	0.0034	0.5000	0.0100
EKF-MA3	1.0382	0.0403	X	X
NON-A-MA4	1.7047	1.9789	X	X
NON-A-MA5	1.3421	1.5717	X	X

Simulations: Analysis

- NON-A algorithms require less hypothesis than EKF (easier to implement),
- All results MA1-BIS, MA2 and MA3 are are better than EKF,
- MA4 and MA5 are not so usefull even if better than EKF (with the same outputs).

Experimental Results: settings

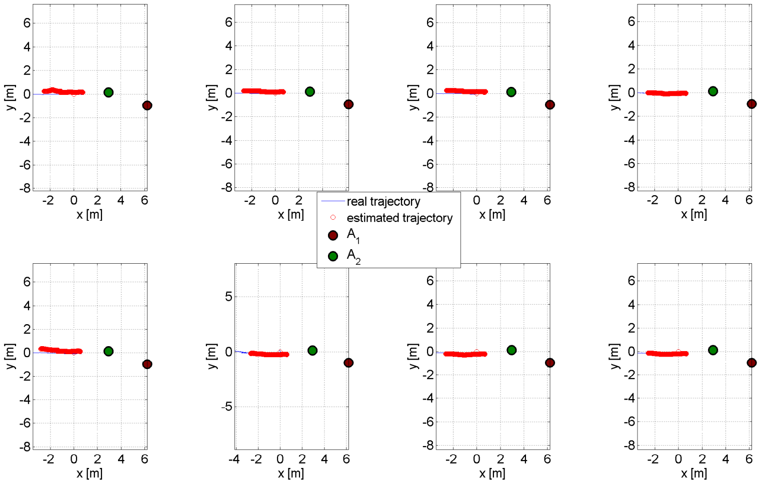
- Robot is equipped with an Imaging Source camera and an inertial sensor for MA2,
- Reference localization is obtained using luminous pattern hanging from the ceiling,
- The Imaging Source camera is used to get the relative angle between the robot and two points (landmark and/or target),
- This measures are process using Matlab to estimate the posture.

Experimental Results: MA1-BIS

	error mean (cm)	error standard deviation (cm)
NON-A- MA1-BIS	11	5

Table: Experimental results of NON-A algorithm

Experimental Results: MA1-BIS



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Conclusion

- Localization framework: bearing only measurement cases using optic flow information and natural landmarks.
- Localizability is defined in a differential algebraic framework (notion of localizability defect)
- Localization $\Leftrightarrow$ numerical differentiation problem in noisy environment.
- Our solution provides pose and velocities reconstruction.

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