Controllability and Motion Planning for 2-D (and 3-D) vehicles formations

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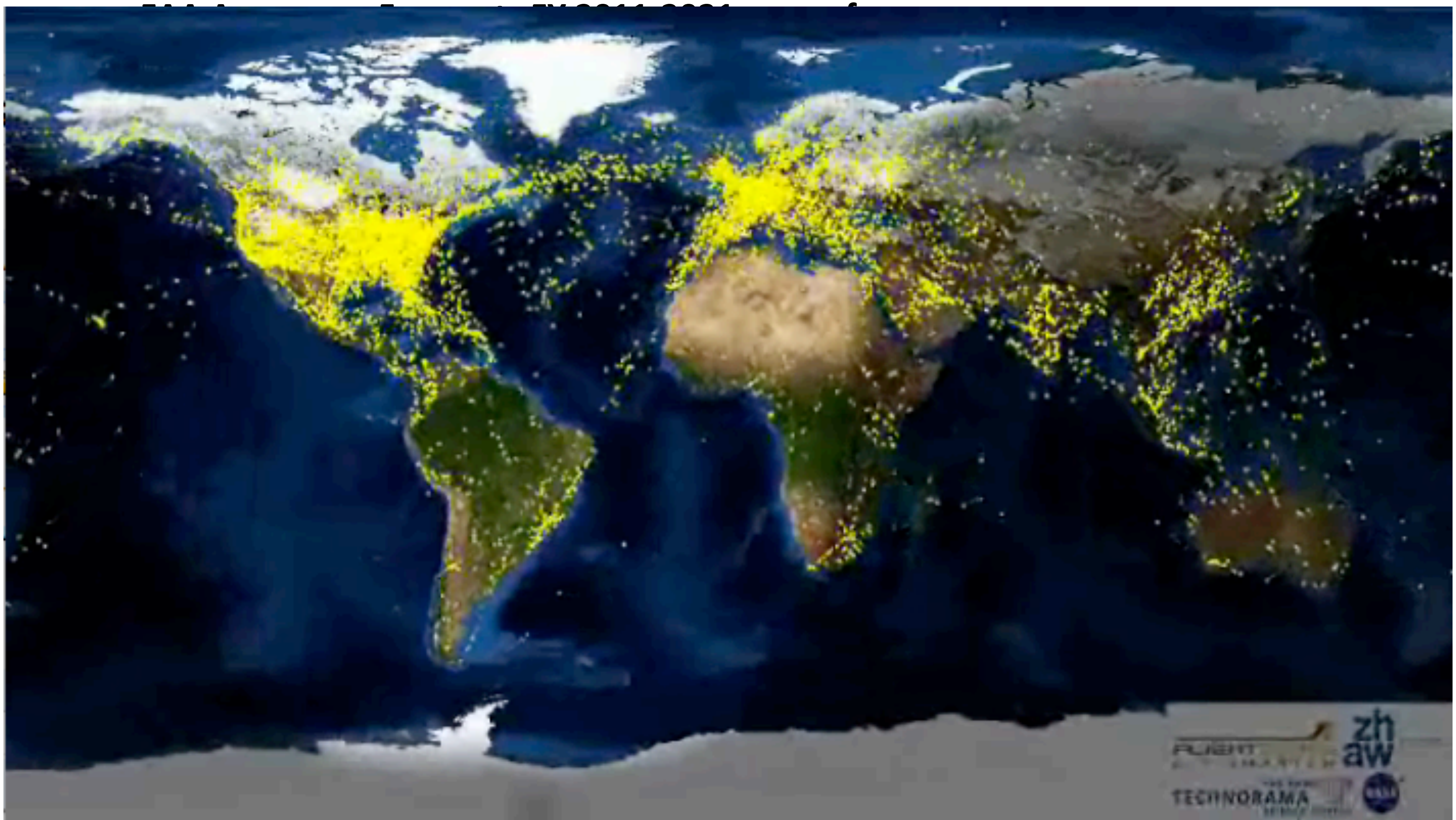
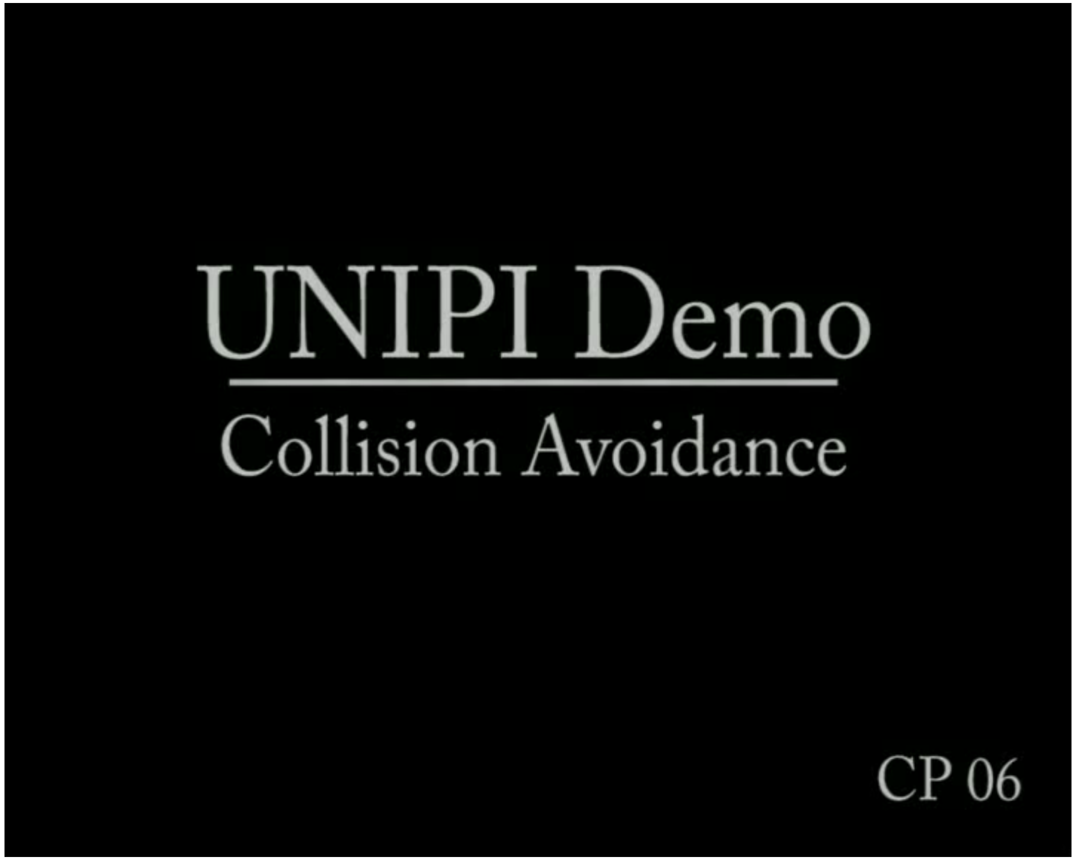
Coordination in a Society of Robots

Coordination in real environments

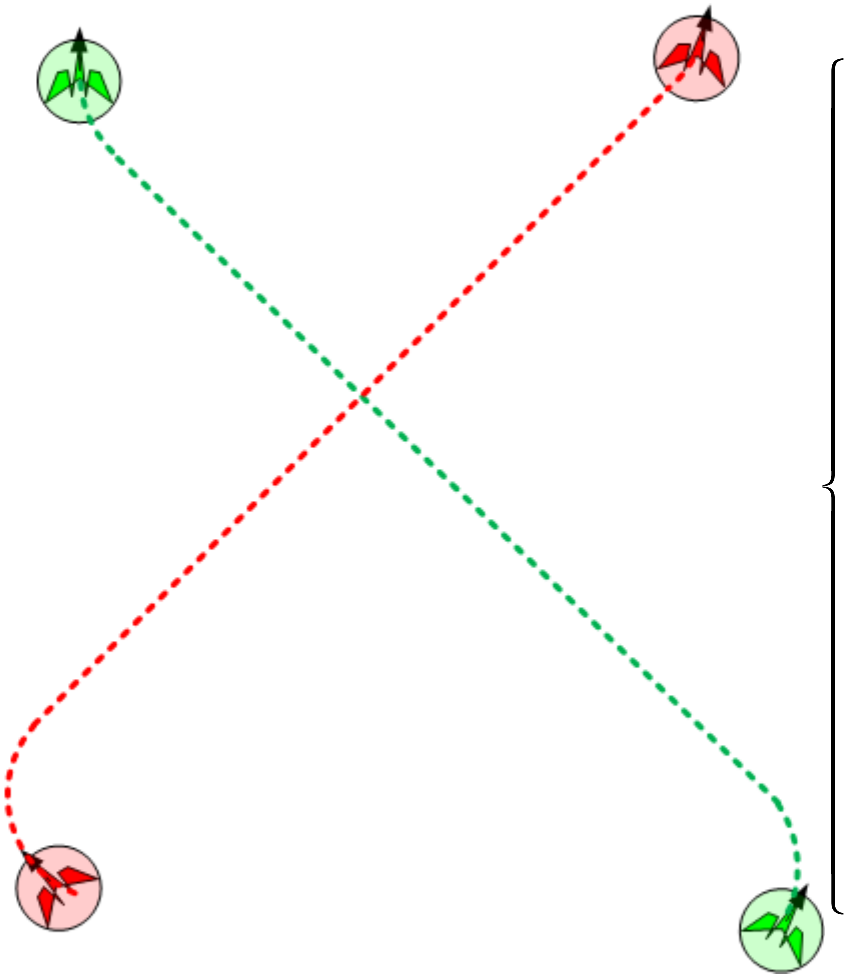


commercial Unmanned Aircraft Systems in five years.

Optimal safe coordination



Optimal safe coordination

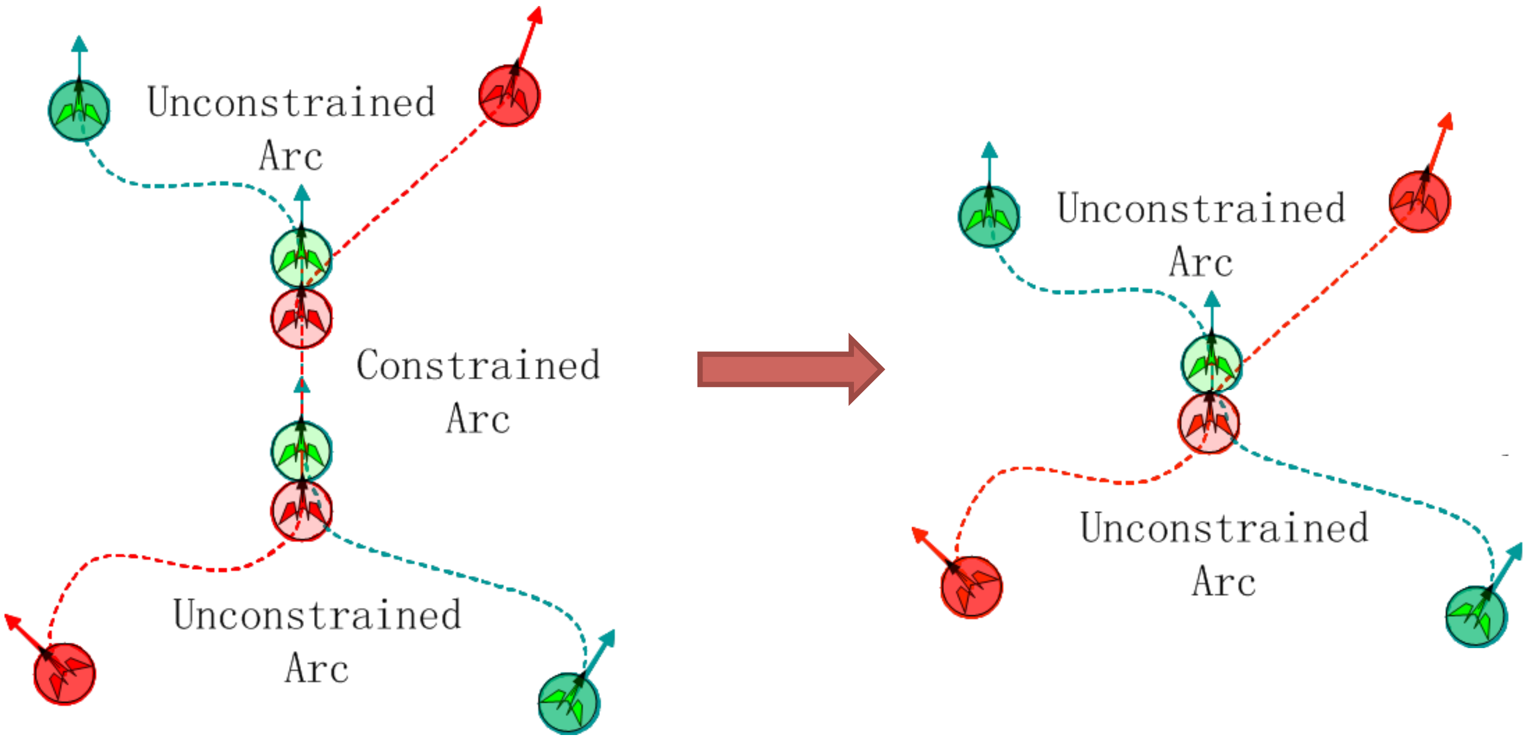


$$\begin{cases} \min J \\ \dot{q}_i = \begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{pmatrix} = \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \\ \omega_i \end{pmatrix} & i = 1,\ldots,N \\ |\omega_i| \le \Omega & i = 1,\ldots,N \\ D^2_{i,j}(t) \ge d^2_{i,j} & \forall t, \quad i,j = 1,\ldots,N \\ q_i(T_i^s) = q_i^s, q_i(T_i^f) = q_i^f & i = 1,\ldots,N \\ D^2_{i,j}(t) = (x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2 \end{cases}$$

SOCIAL BEHAVIOUR

Coordinative: Aware of others, Individual Goals, Actions not helpful to other robots

Optimal solution



Formations

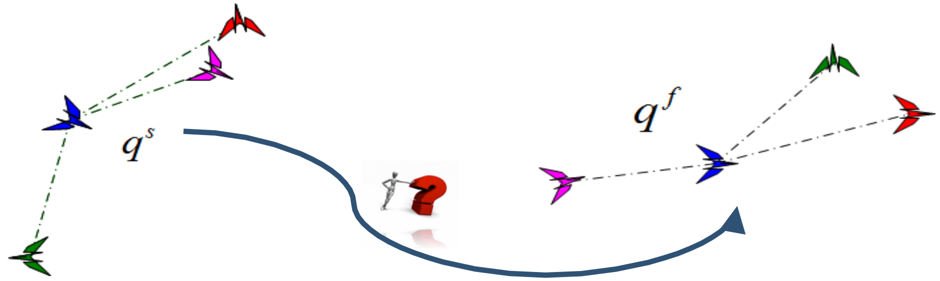
SOCIAL BEHAVIOUR

Cooperative: Aware of teammates, Shared Goals, Actions beneficial to teammates

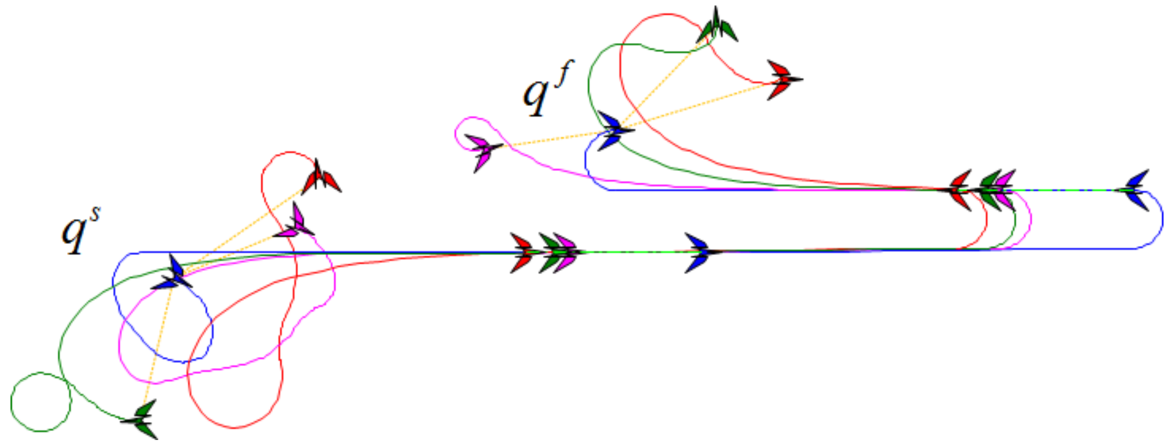
Optimal paths for formations

Towards shortest path for non-holonomic vehicles in formation:

- Controllability

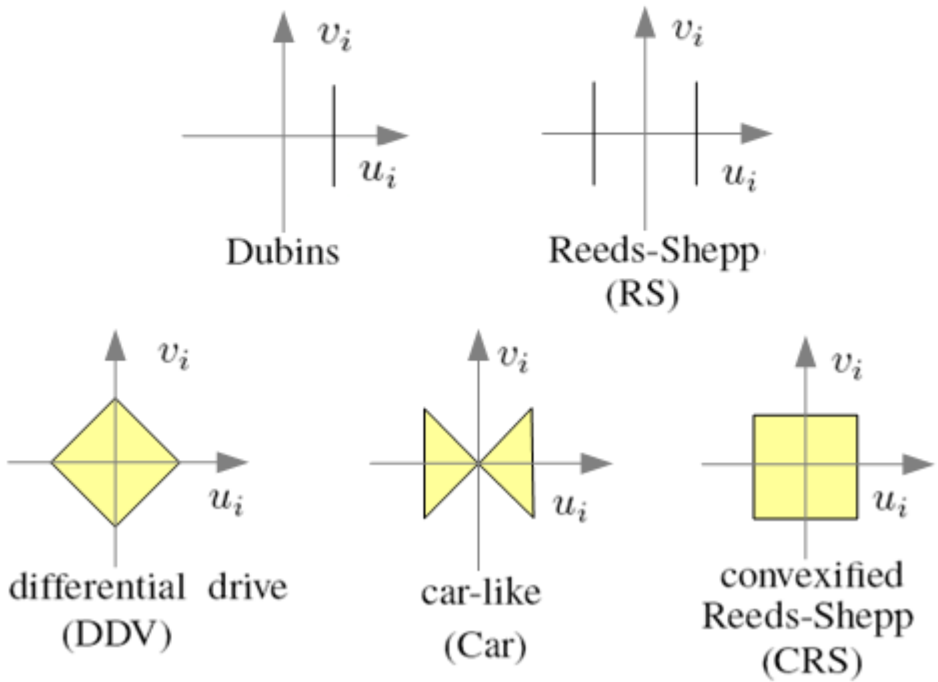


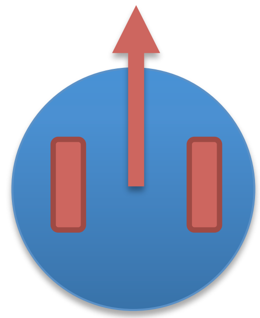
- Motion Planning

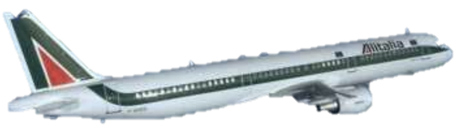


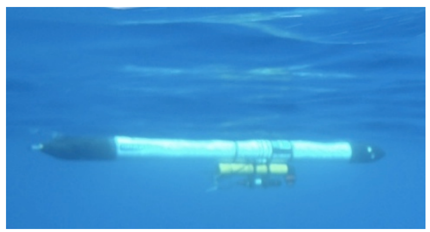
..... Optimal paths: on going research!

Which robots?



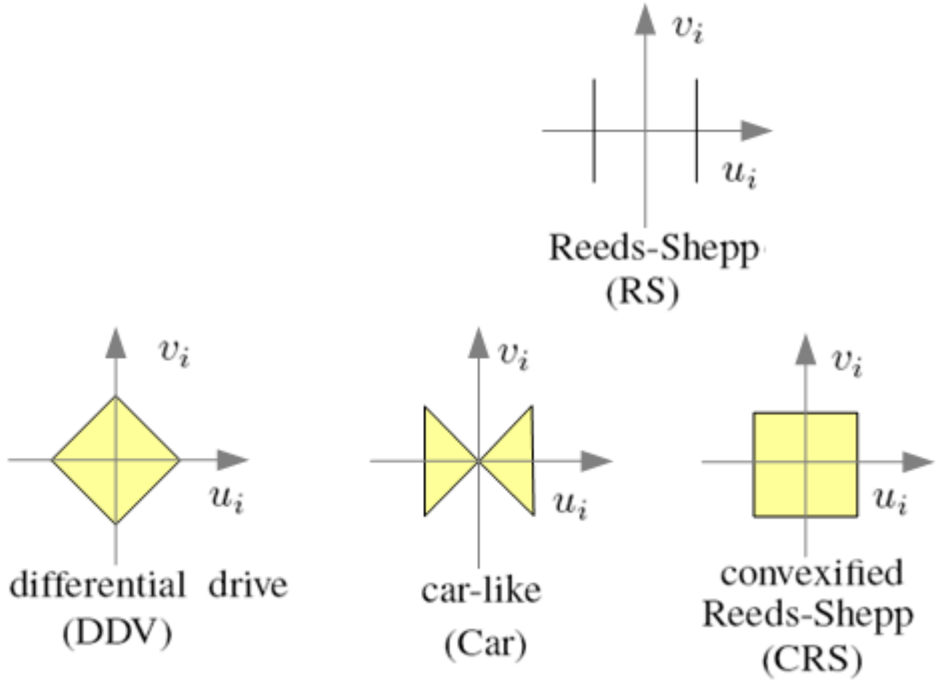


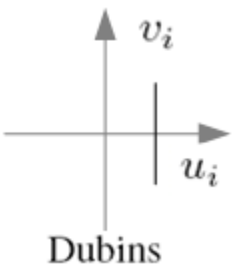




Controllability

Controllability results for four types of pairs of robot vehicles





Controllability for pairs of Dubins ➔ controllability for formations of Dubins

ICRA2010, "Controllability for Pairs of Vehicles Maintaining Constant Distance" H. Wang, L. Pallottino, A.Bicchi

Dubins Vehicles Case

- Configuration of a pair of identical vehicles

$$\xi = \left(x_1, y_1, \theta_1, x_2, y_2, \theta_2\right)$$

- Kinematic model of the system is:

$$\dot{\xi} = \left(\cos(\theta_1), \sin(\theta_1), v_1, \sin(\theta_2), \cos(\theta_2), v_2\right)$$

- subject to the constant distance constraint

$$d(t) = (y_2(t) - y_1(t))^2 + (x_2(t) - x_1(t))^2 = D^2$$

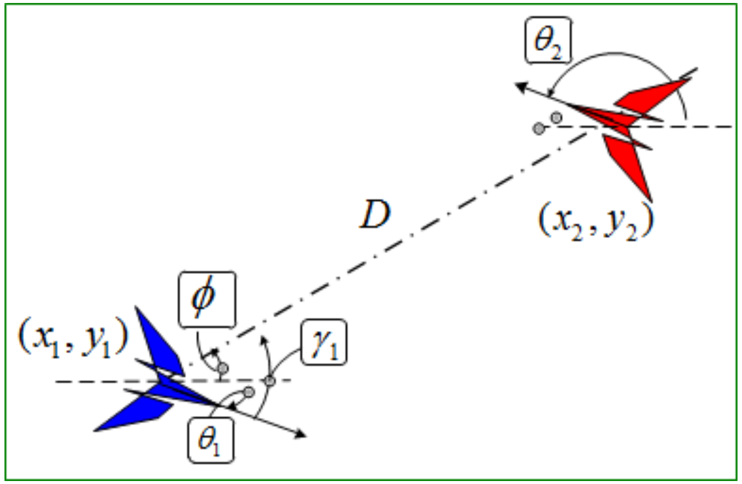
- Control:

$$v_i \in U = \left[-\frac{1}{R}, \frac{1}{R}\right]$$

Dubins Vehicles Case

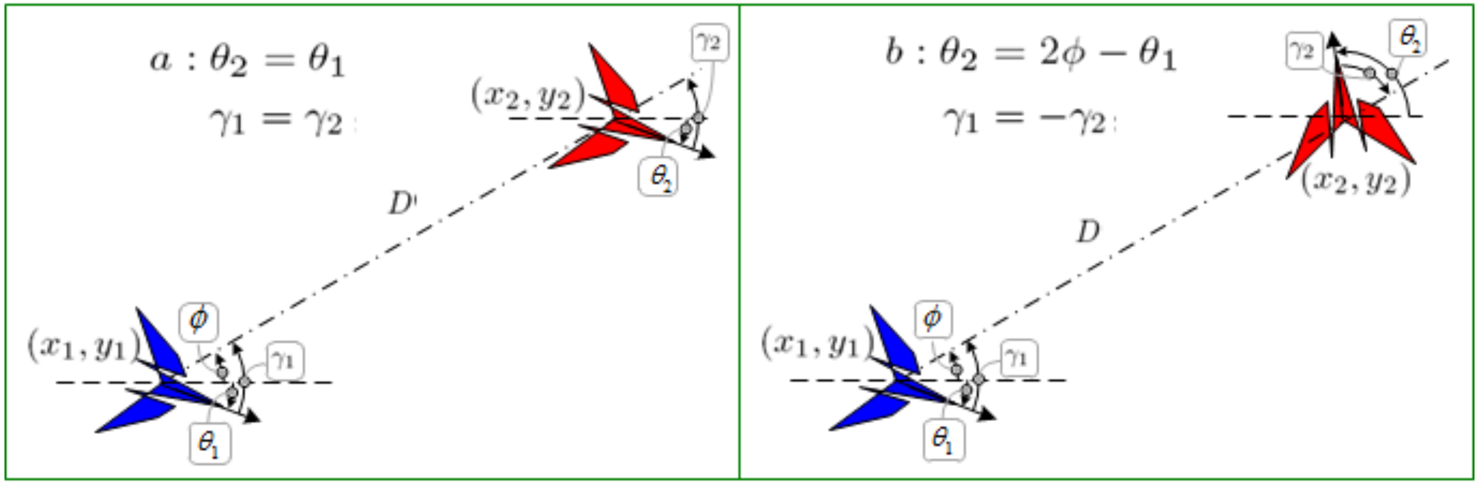
- New configuration

$$q = (x_1, y_1, \theta_1, \phi, \theta_2)$$

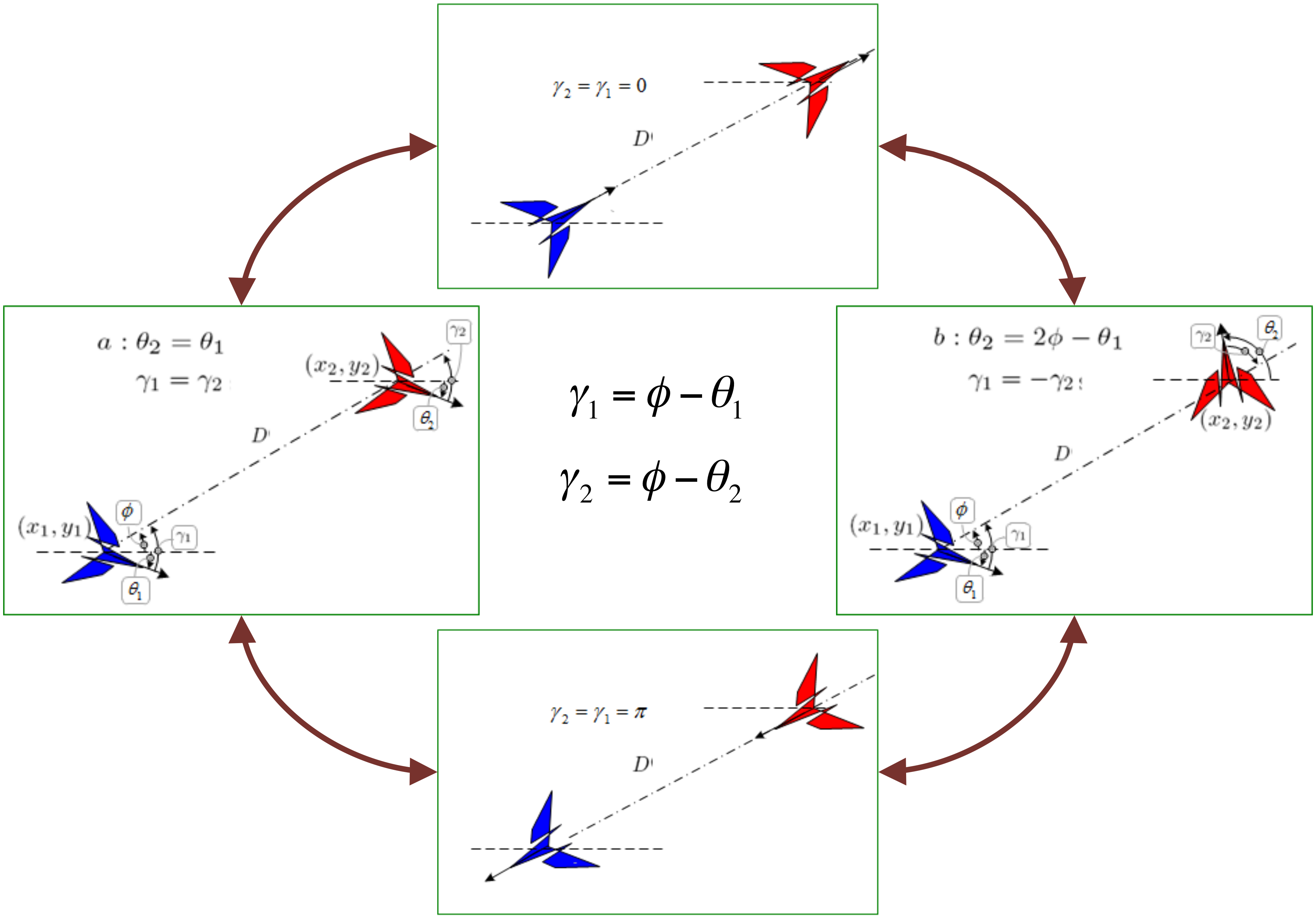


$$d(t) \equiv D \Rightarrow \dot{d}(t) \equiv 0 \Rightarrow$$

- $a : \theta_1 = \theta_2 \quad b : \theta_1 + \theta_2 = 2\phi$



Dubins Vehicles Case



$$\gamma_1 = \phi - \theta_1$$

$$\gamma_2 = \phi - \theta_2$$

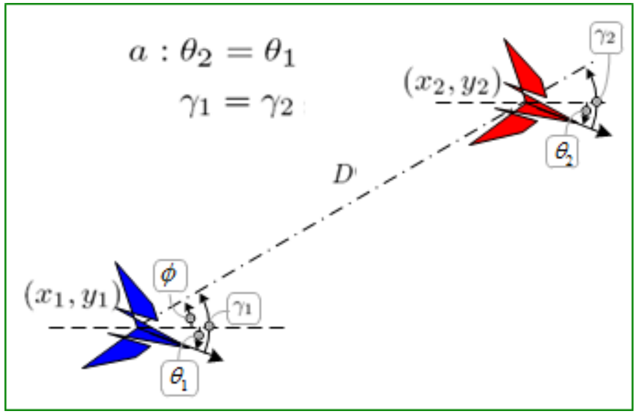
Parallel Case

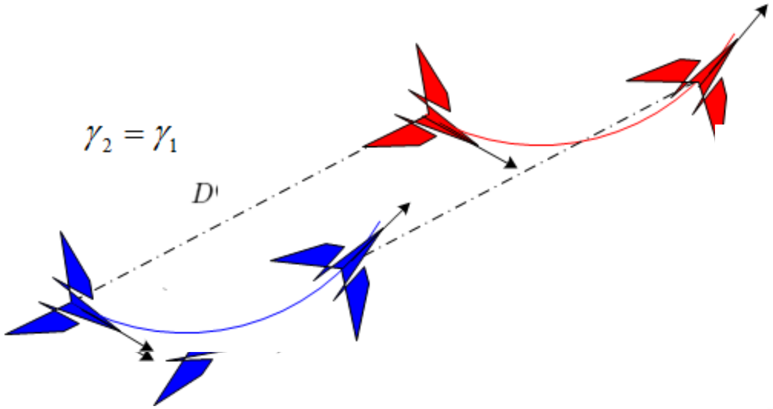
$$q^a = (x_1, y_1, \theta_1, \phi)$$

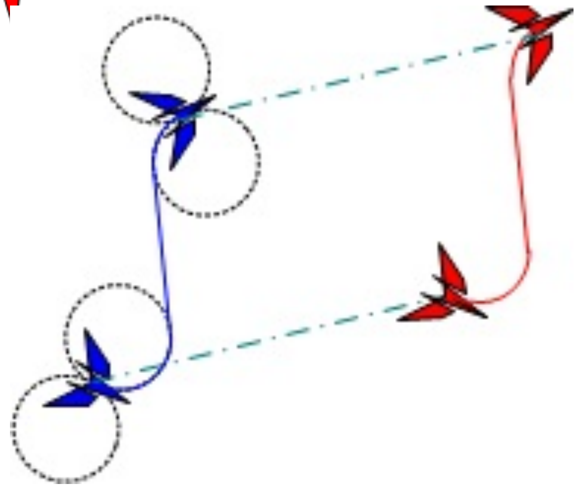
$$\dot{q}^a = (\cos(\theta_1), \sin(\theta_1), v_1, 0)$$

$$\phi(t) = const.$$

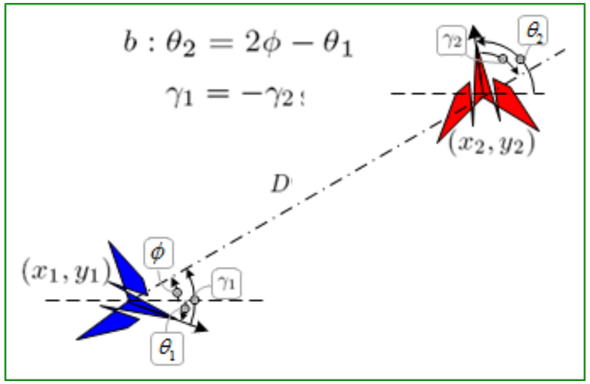
$$v_1 = v_2$$







Symmetric Case

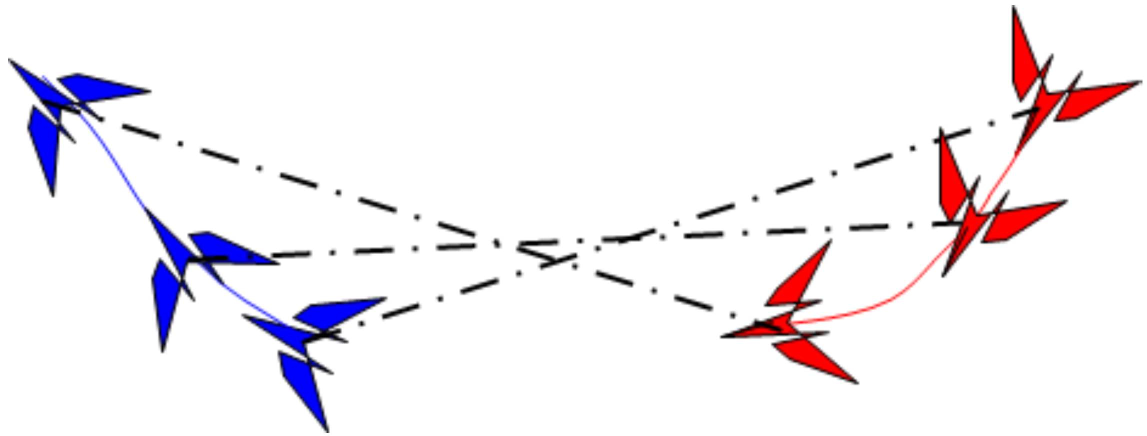


$$q^b = \left(x_1, y_1, \theta_1, \gamma_1\right)$$

$$\dot{\phi} = \frac{2}{D}\sin(\phi - \theta_1) \quad \dot{\gamma}_1 = \frac{2}{D}\sin(\gamma_1) - v_1$$

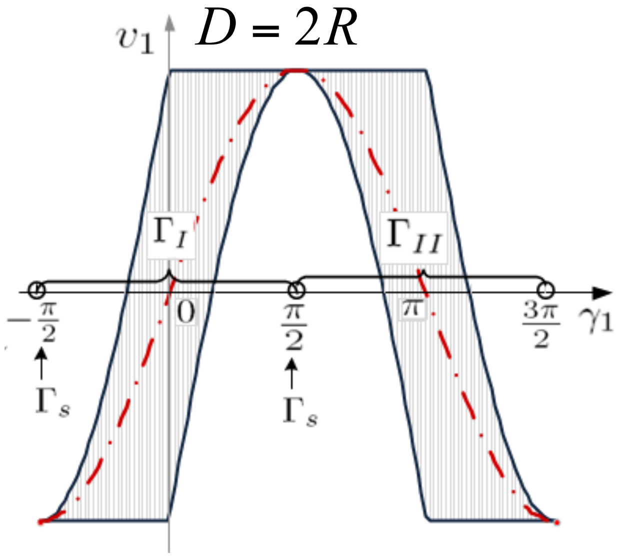
$$v_1 = \frac{4}{D}\sin(\phi - \theta_1) - v_2 = \frac{4}{D}\sin(\gamma_1) - v_2$$

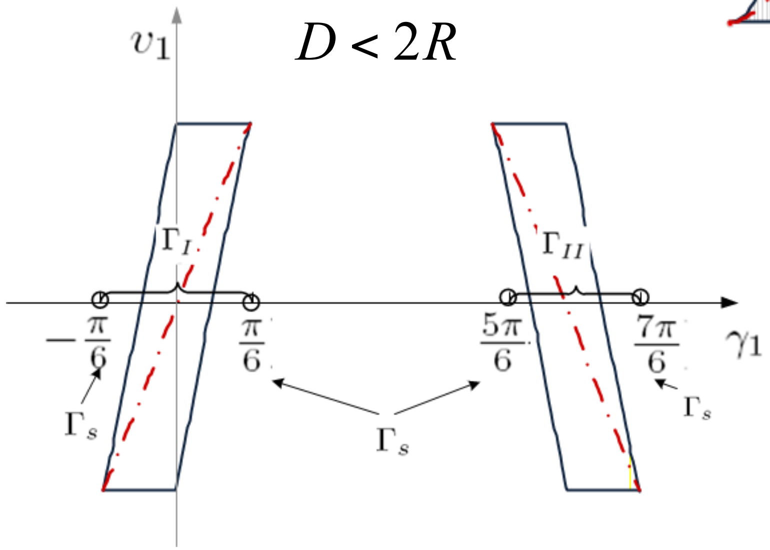
$$v_1 \in U_1 = \left[-\frac{1}{R} + \max\left\{\frac{4}{D}\sin(\gamma_1), 0\right\}, \frac{1}{R} + \min\left\{\frac{4}{D}\sin(\gamma_1), 0\right\}\right]$$

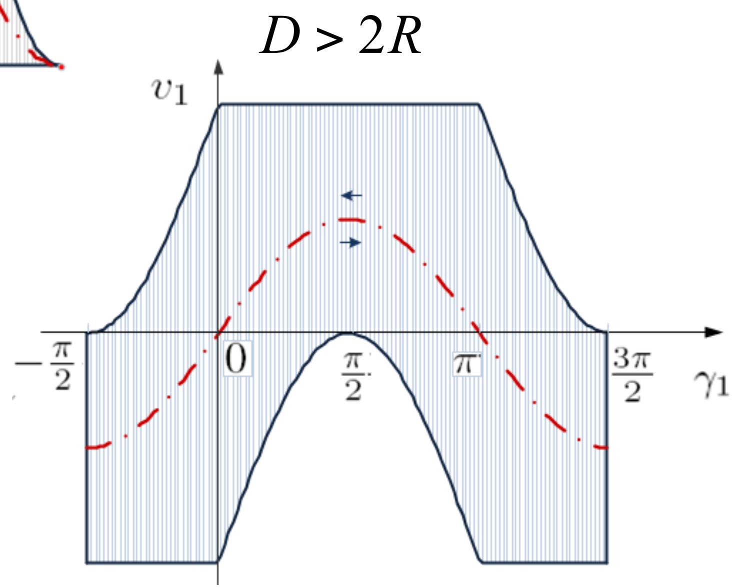


Control Space

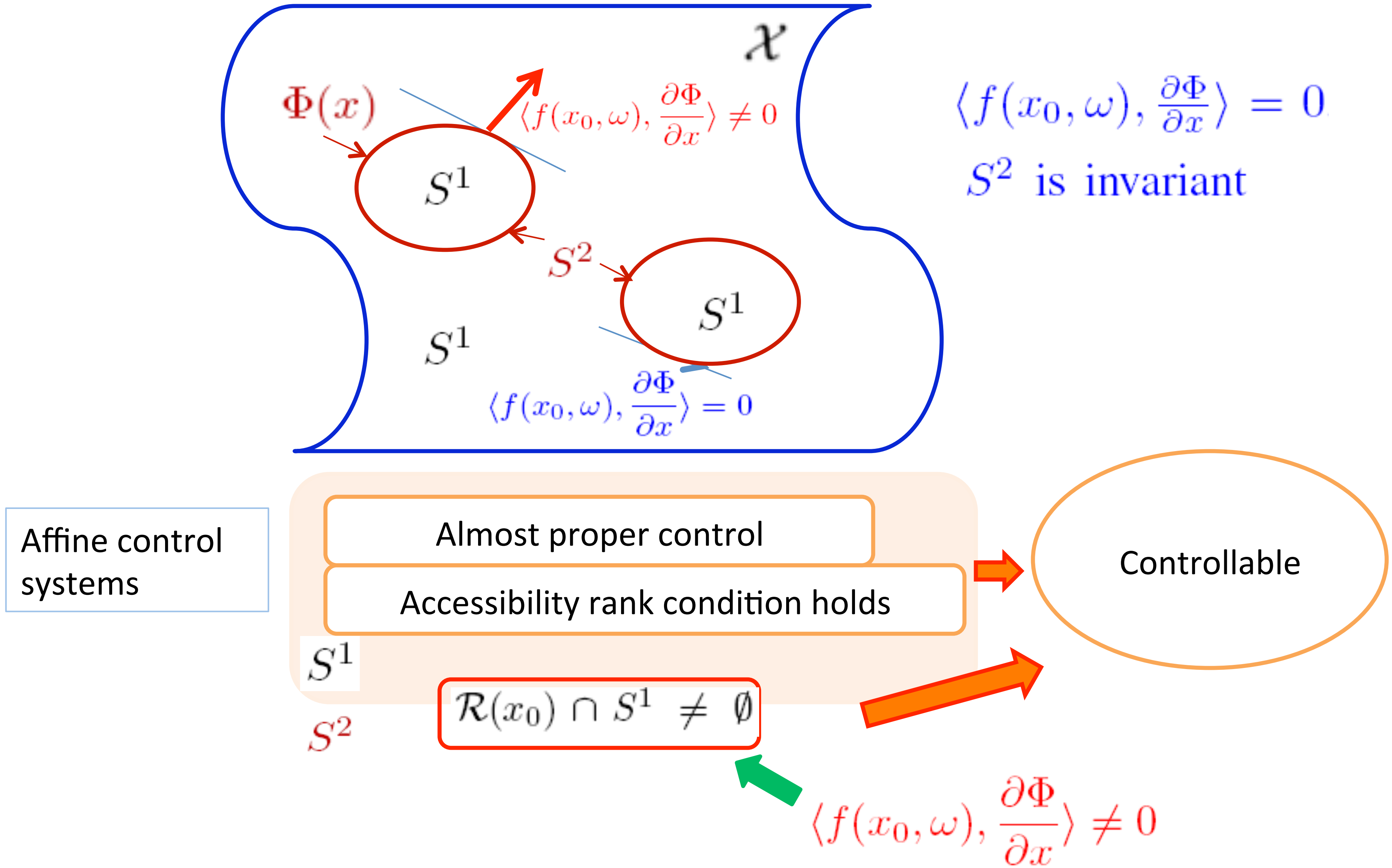
$$v_1 \in U_1 = \left[-\frac{1}{R} + \max\left\{ \frac{4}{D}\sin(\gamma_1), 0 \right\}, \frac{1}{R} + \min\left\{ \frac{4}{D}\sin(\gamma_1), 0 \right\} \right]$$



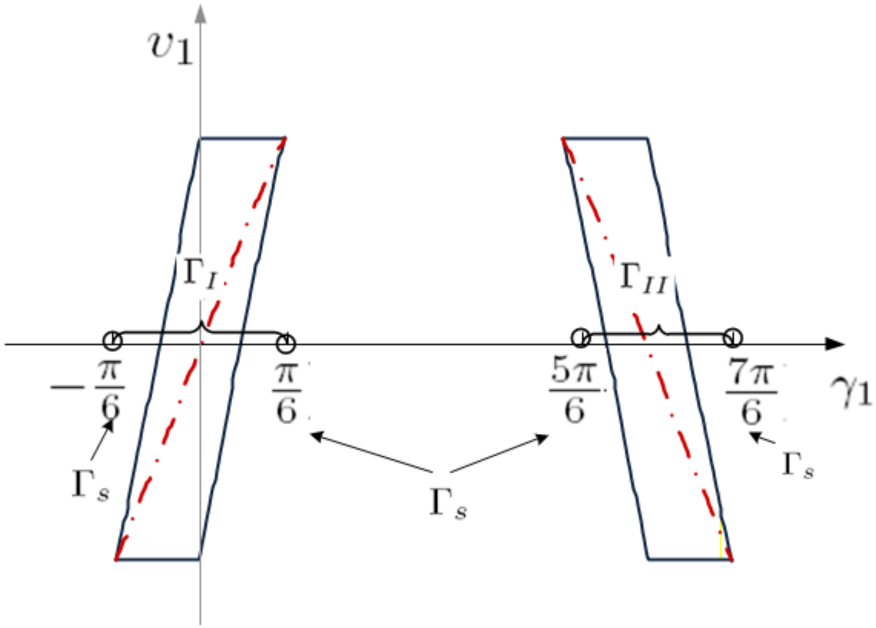


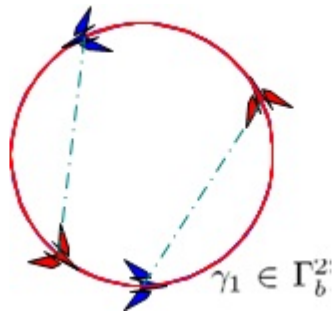


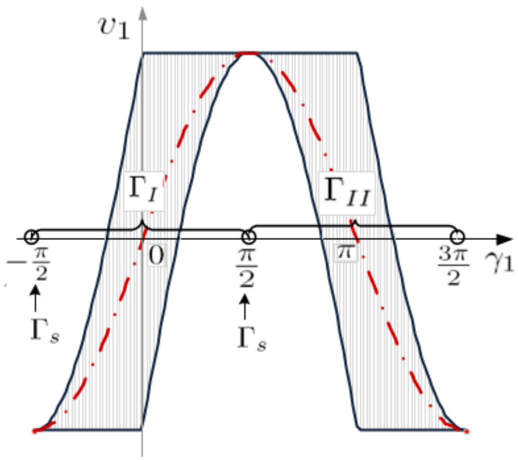
Controllability Theorems – Main Idea

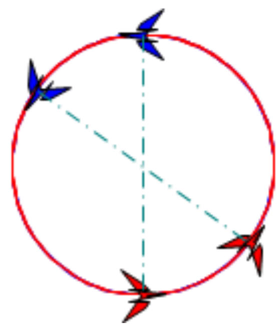


Controllability for D<=2R

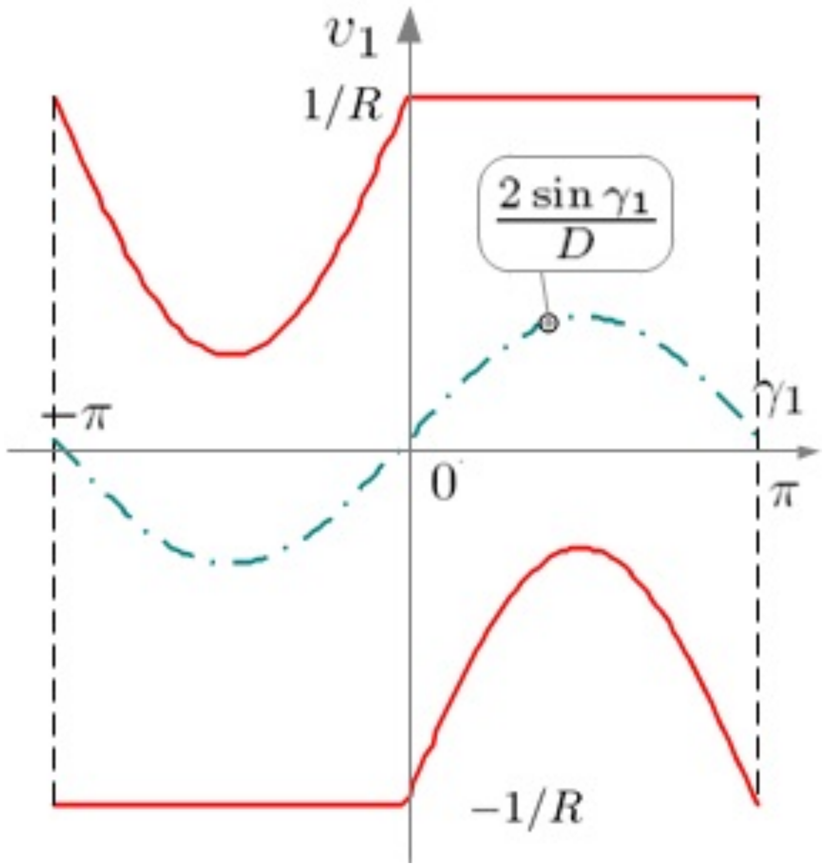




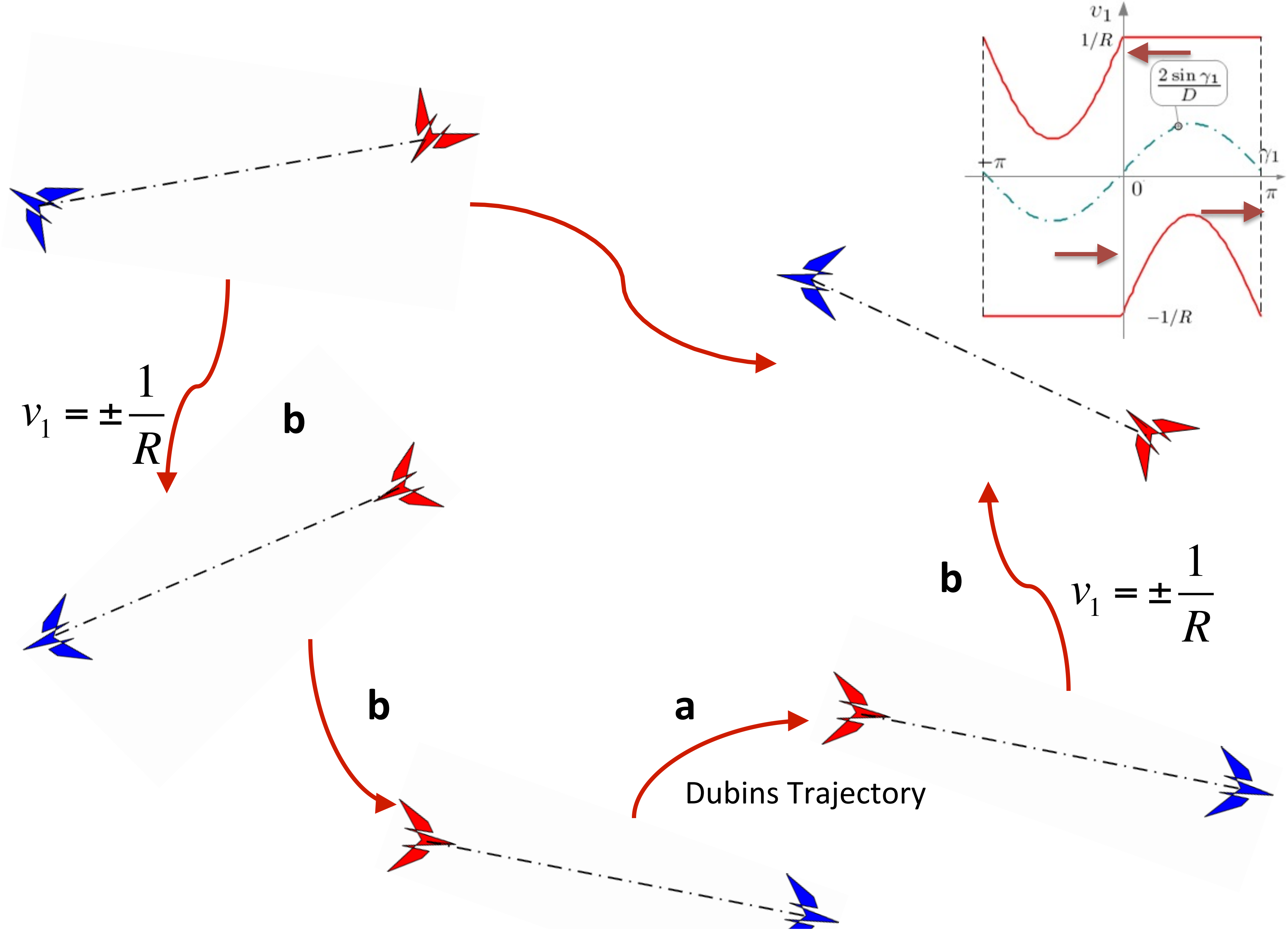




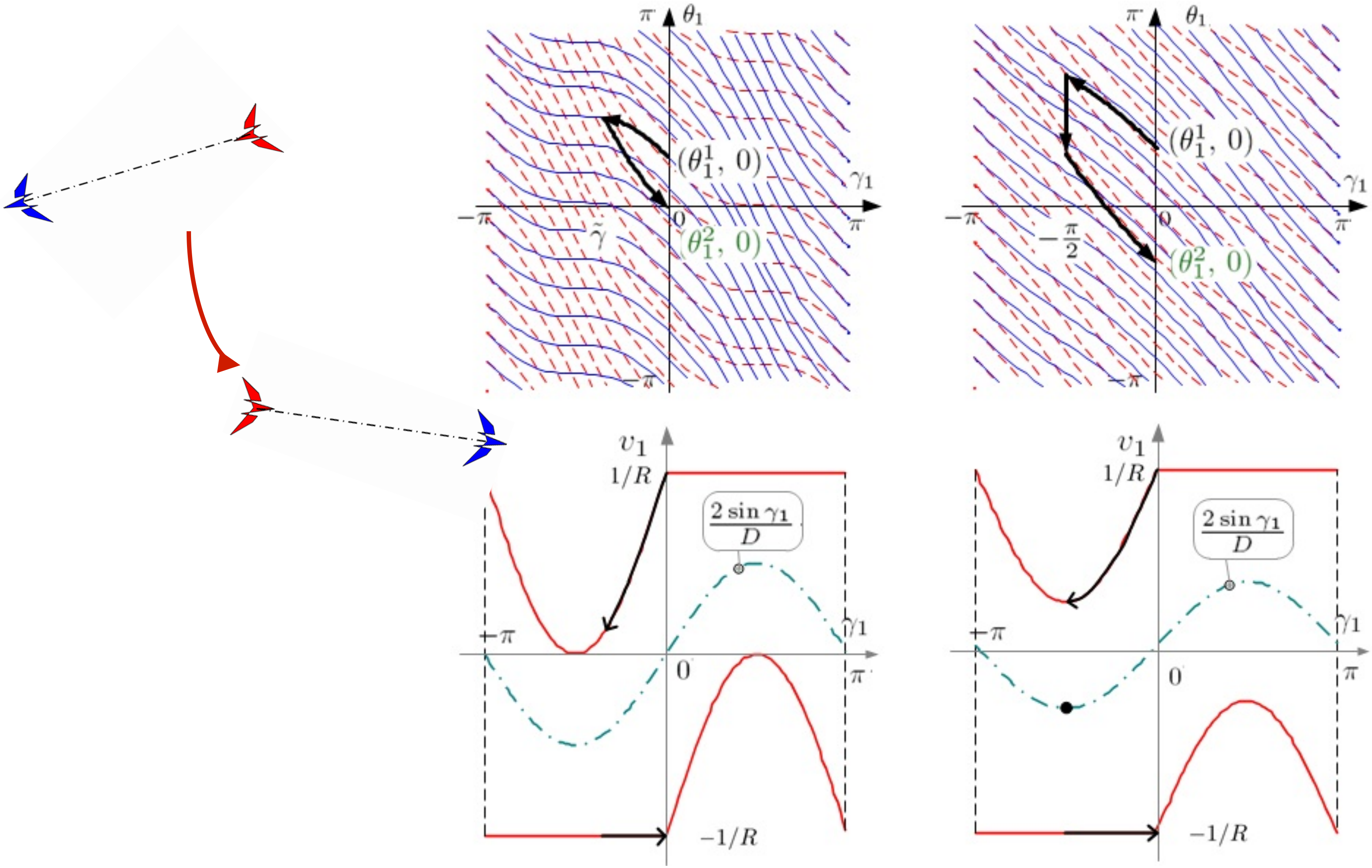
Controllability for D>2R



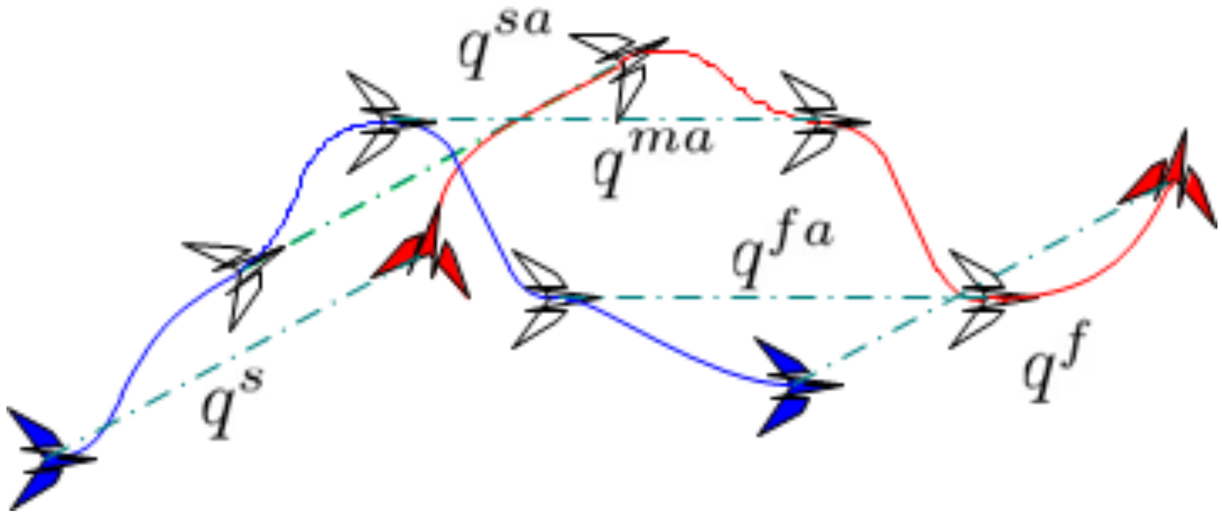
Motion Planning for pairs of Dubins



Motion Planning for pairs of Dubins

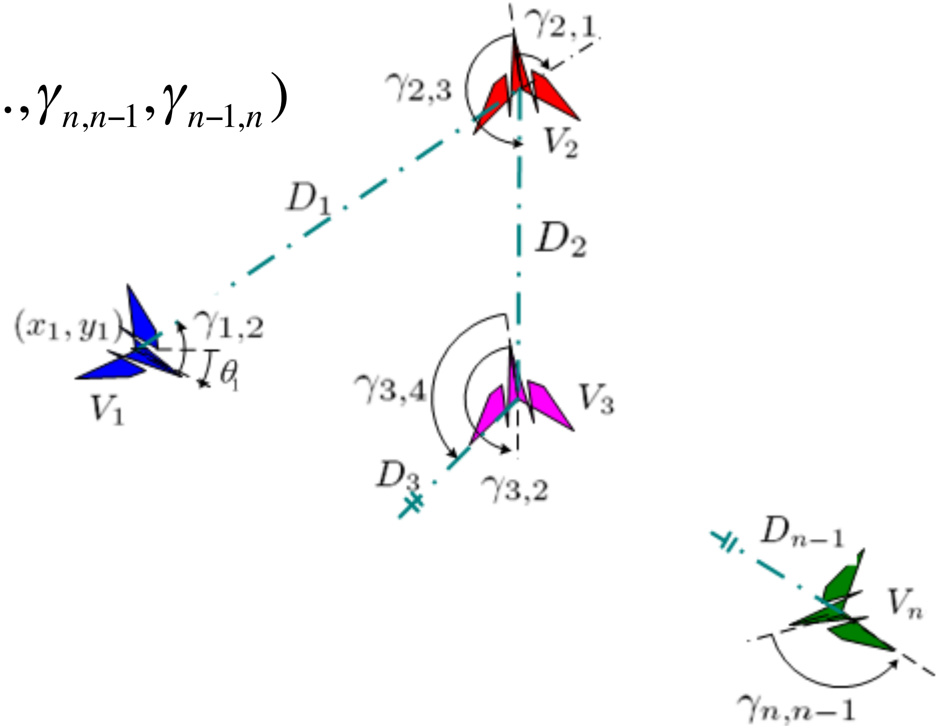


Motion Planning for pairs of Dubins



Chain Formations

$$q = (x_1, y_1, \theta_1, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,3}, \gamma_{3,2}, \ldots, \gamma_{n,n-1}, \gamma_{n-1,n})$$



$$\dot{q} = g_0^c + \sum_{i=1}^{n} g_i^c v_i$$

$$g_0^c = (\cos\theta_1, \sin\theta_1, 0, \frac{\sin\gamma_{1,2} - \sin\gamma_{2,1}}{D_1}, \frac{\sin\gamma_{1,2} - \sin\gamma_{2,1}}{D_1}, \ldots, \frac{\sin\gamma_{n-1,n} - \sin\gamma_{n,n-1}}{D_{n-1}}, \frac{\sin\gamma_{n-1,n} - \sin\gamma_{n,n-1}}{D_{n-1}})^T$$

$$g_1^c = e_3 - e_4$$

$$g_i^c = -e_{2i} - e_{2i+1}, i = 2, \ldots, n-1 \qquad e_i = \left(0, \ldots, 0, \underbrace{1}_{i}, 0, \ldots, 0\right) \in \Re^{2n+1}$$

$$g_n^c = -e_{2n+1}$$

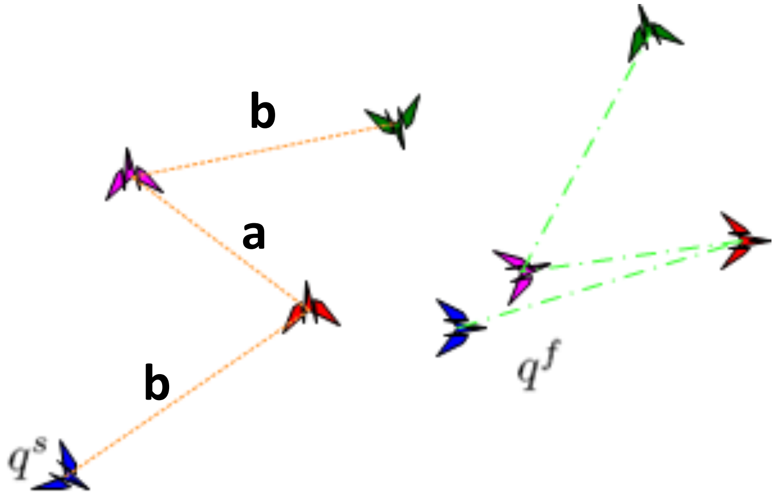
Chain Formations

Theorem:

If

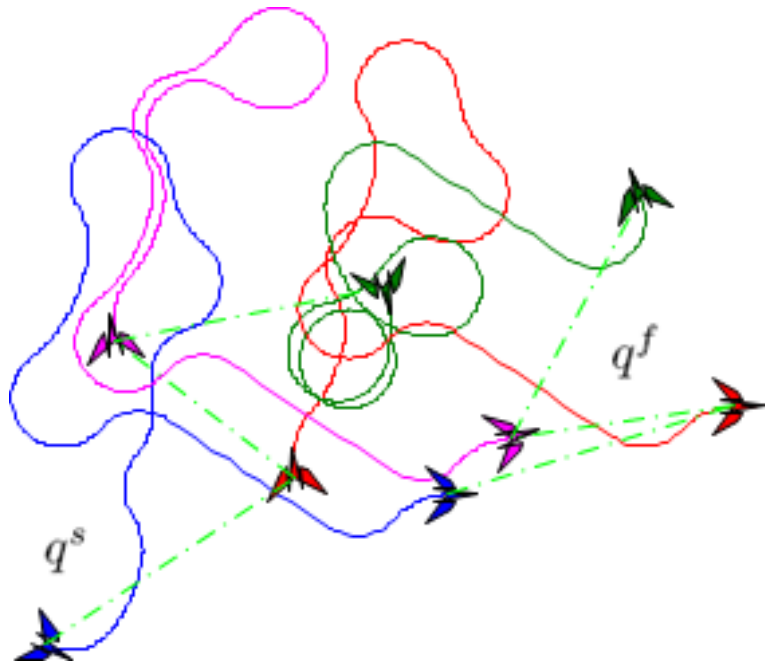
$$\sum_{i=1}^{n-1} \frac{1}{D_i} < \frac{1}{2R}$$

the system is controllable



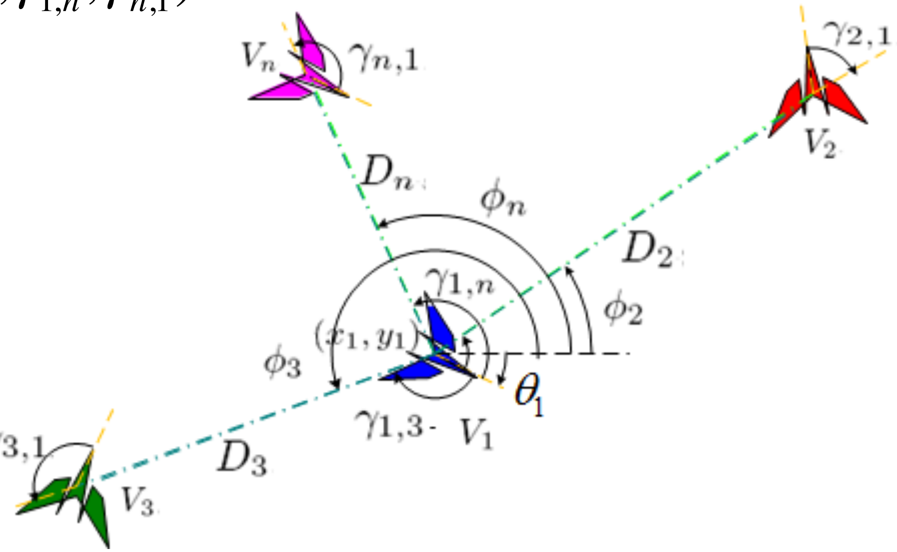
$$a: \mathrm{v}_{i+1} = \mathrm{v}_i$$

$$b: \mathrm{v}_{i+1} = \frac{4}{D_i}\sin(\gamma_{i,i+1}) - \mathrm{v}_i$$



Star Formations

$$q = (x_1, y_1, \theta_1, \gamma_{1,2}, \gamma_{2,1}, \gamma_{1,3}, \gamma_{3,1}, \ldots, \gamma_{1,n}, \gamma_{n,1})$$



$$\dot{q} = g_0^s + \sum_{i=1}^{n} g_i^s v_i$$

$$g_0^s = (\cos\theta_1, \sin\theta_1,\ 0,\ \frac{\sin\gamma_{1,2} - \sin\gamma_{2,1}}{D_2}, \frac{\sin\gamma_{1,2} - \sin\gamma_{2,1}}{D_2}, \ldots, \frac{\sin\gamma_{1,n} - \sin\gamma_{n,1}}{D_n}, \frac{\sin\gamma_{1,n} - \sin\gamma_{n,1}}{D_n})^T$$

$$g_1^s = e_3 - \sum_{i=2}^{n} e_{2i}$$

$$e_i = \left(0, \ldots, 0, \underset{i}{\underbrace{1}}, 0, \ldots, 0\right) \in \Re^{2n+1}$$

$$g_i^s = -e_{2i+1}, i = 2, \ldots, n-1$$

$$g_n^s = -e_{2n+1}$$

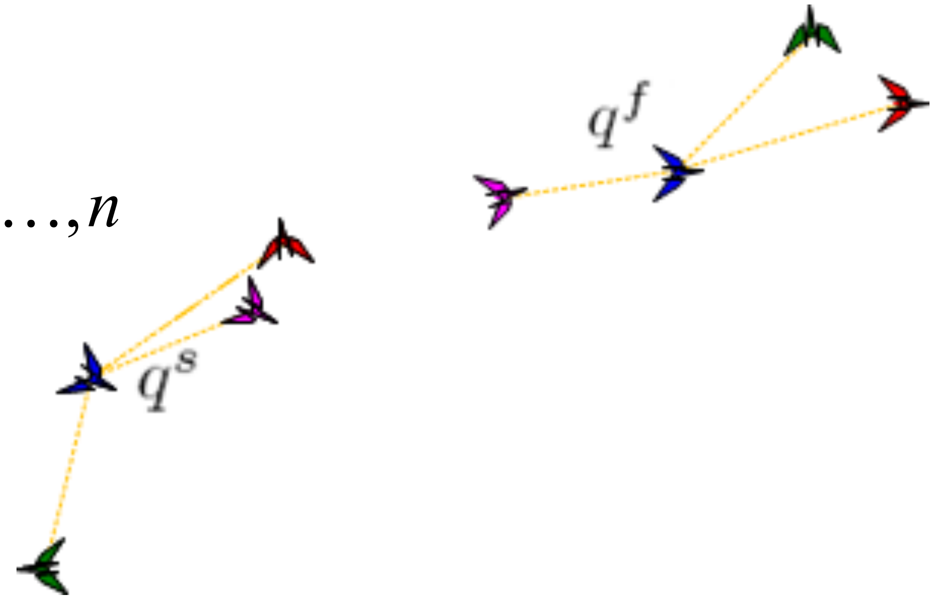
Star Formations

Theorem:

If

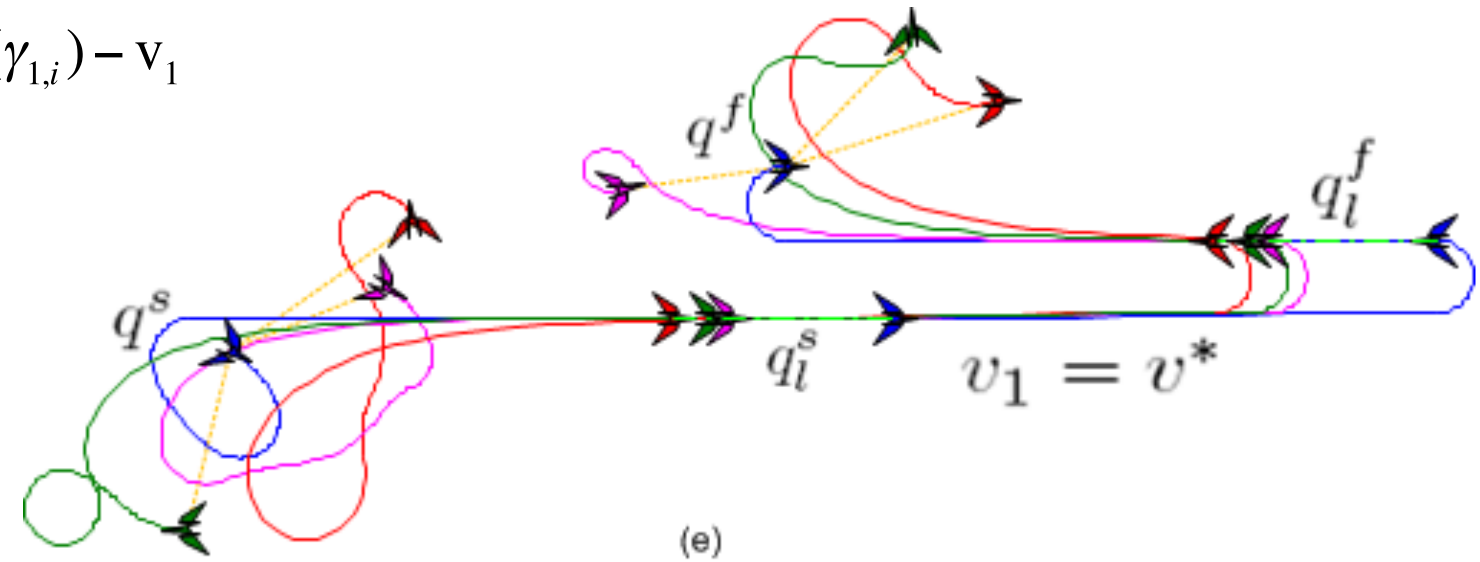
$$\frac{1}{D_i} + \frac{1}{D_j} < \frac{1}{2R}, \forall i \neq j = 1, \ldots, n$$

the system is controllable



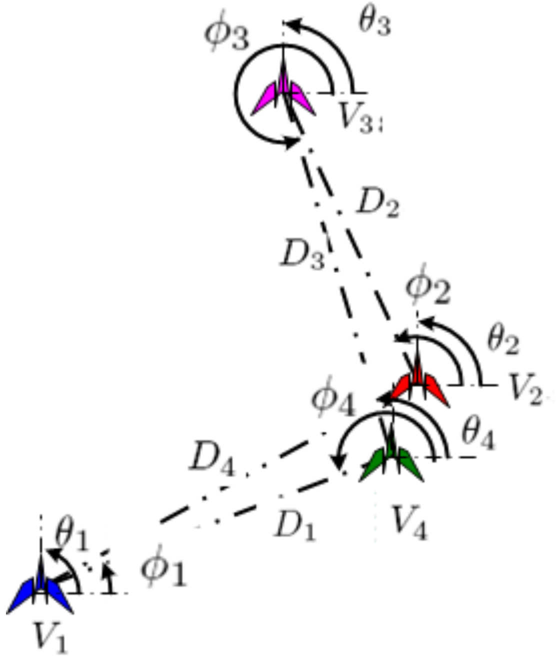
$$a: \mathrm{v}_i = \mathrm{v}_1$$

$$b: \mathrm{v}_i = \frac{4}{D_i}\sin(\gamma_{1,i}) - \mathrm{v}_1$$

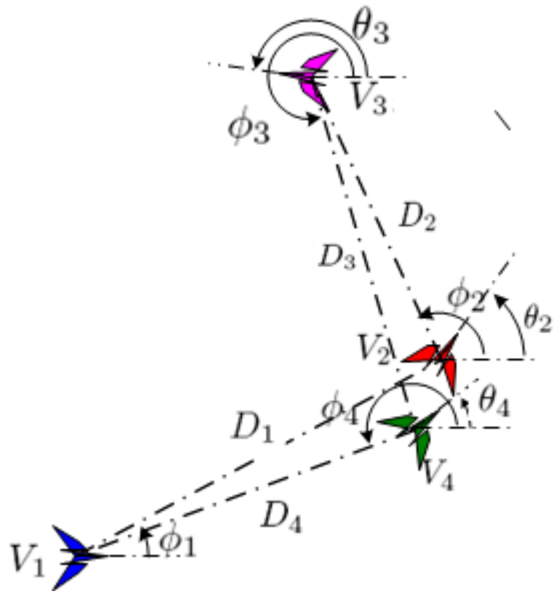


(e)

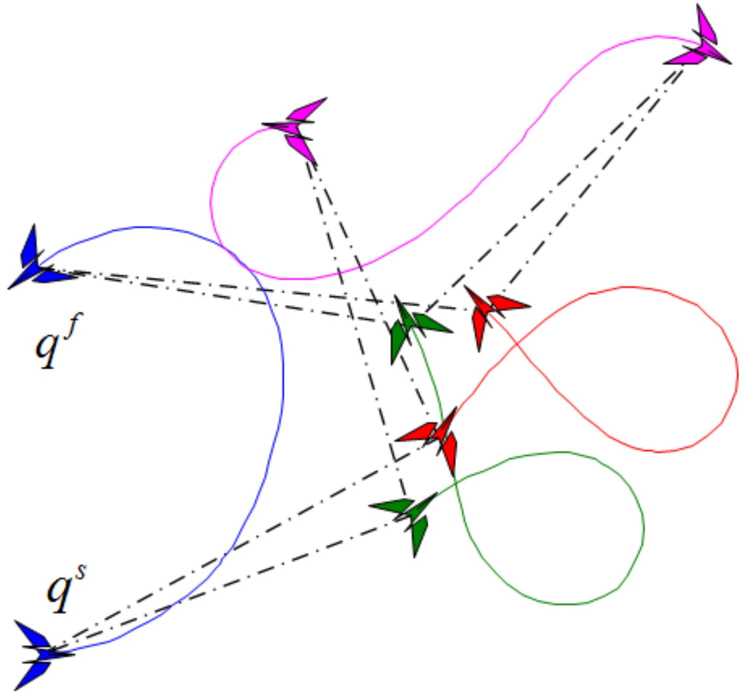
Motion planning for Ring Formations



a type

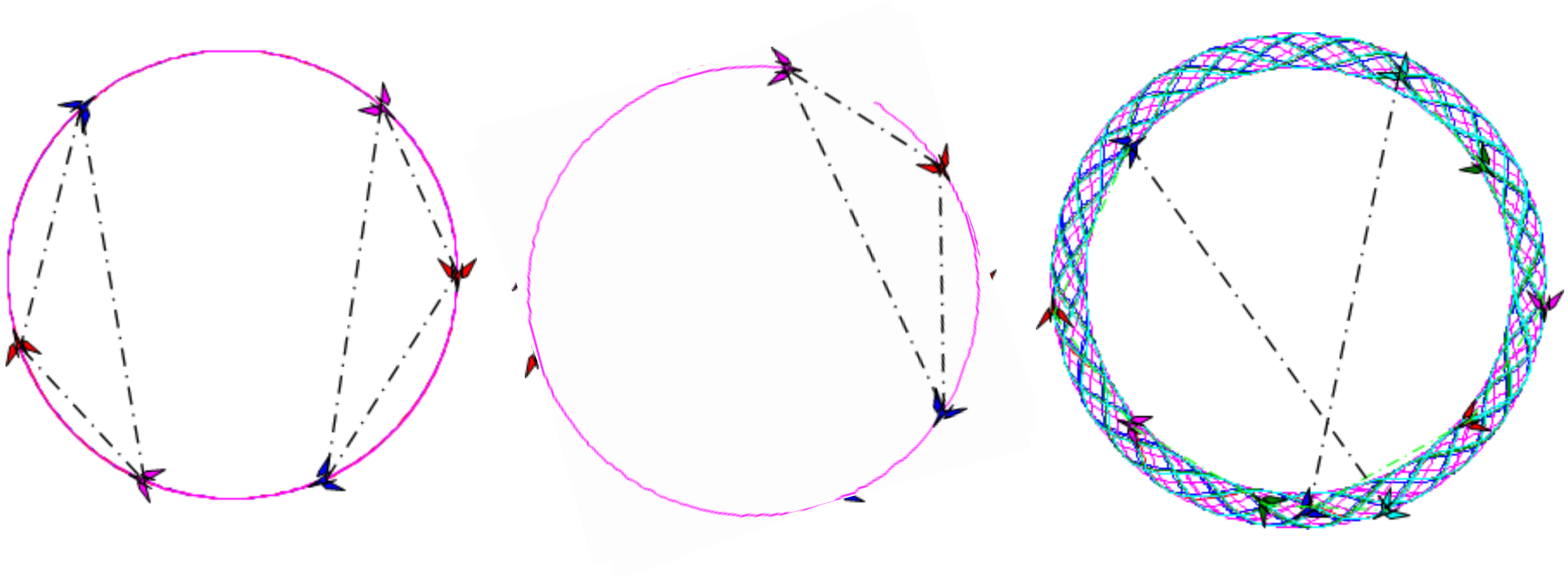


b type



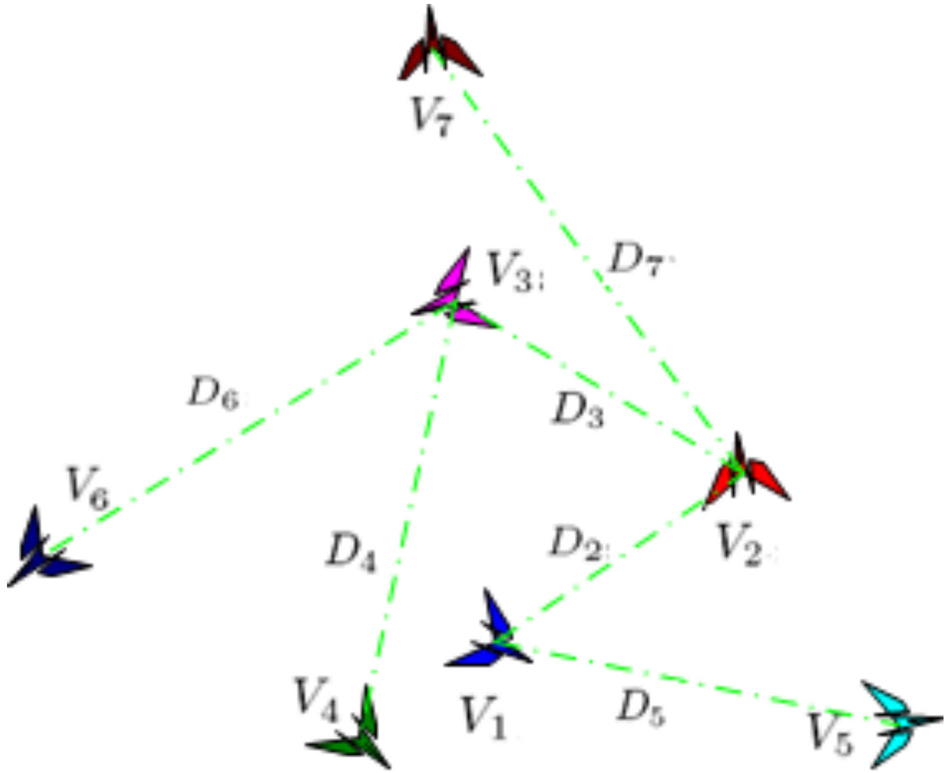
n even

Motion planning for Ring Formations

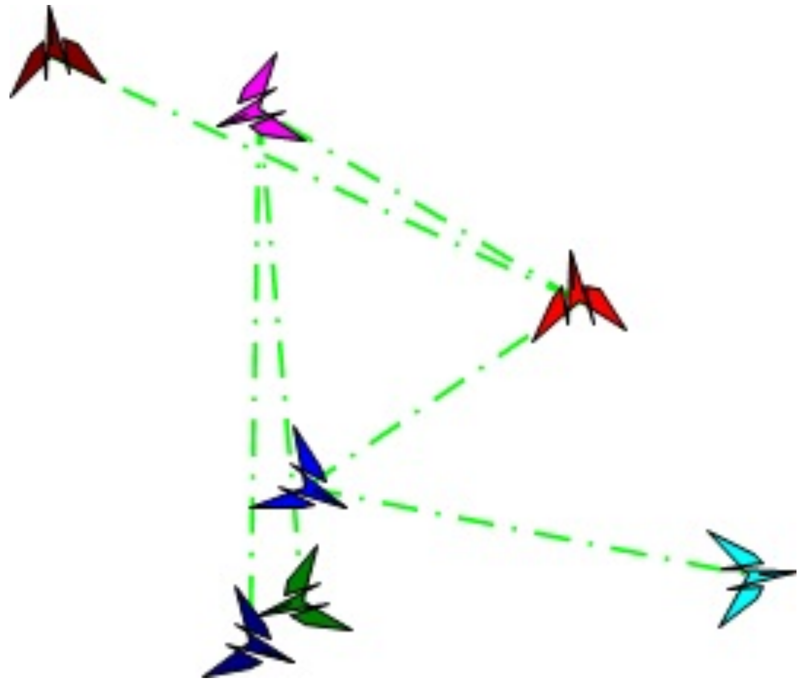


n odd – b type

Tree Formations



Motion Planning for Tree Formations



First step:

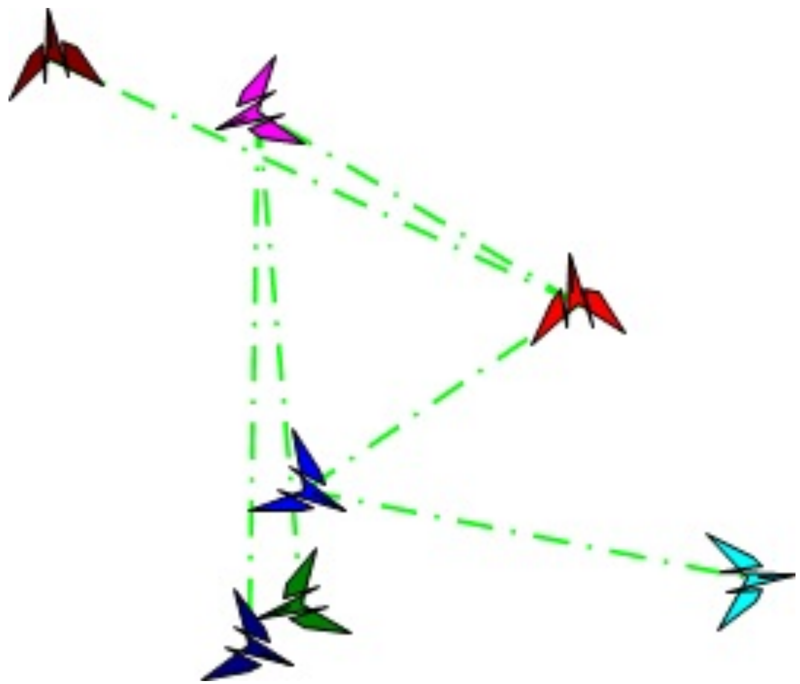
Steer any pair from any **b** to **a** type

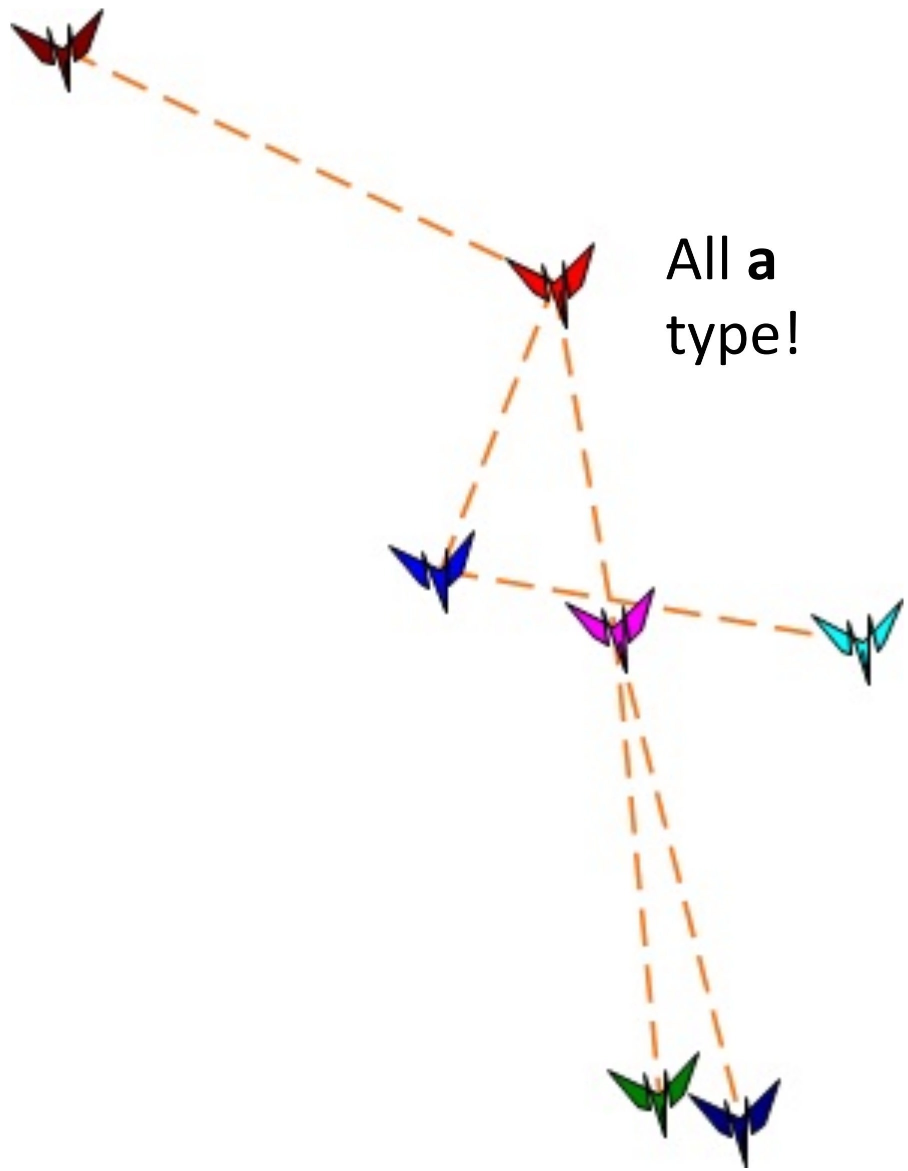
for example: $v_1 = v_{max}$

$a: \mathrm{v}_i = \mathrm{v}_j$

$b: \mathrm{v}_i = \frac{4}{D_{i,j}} \sin(\gamma_{i,j}) - \mathrm{v}_j$

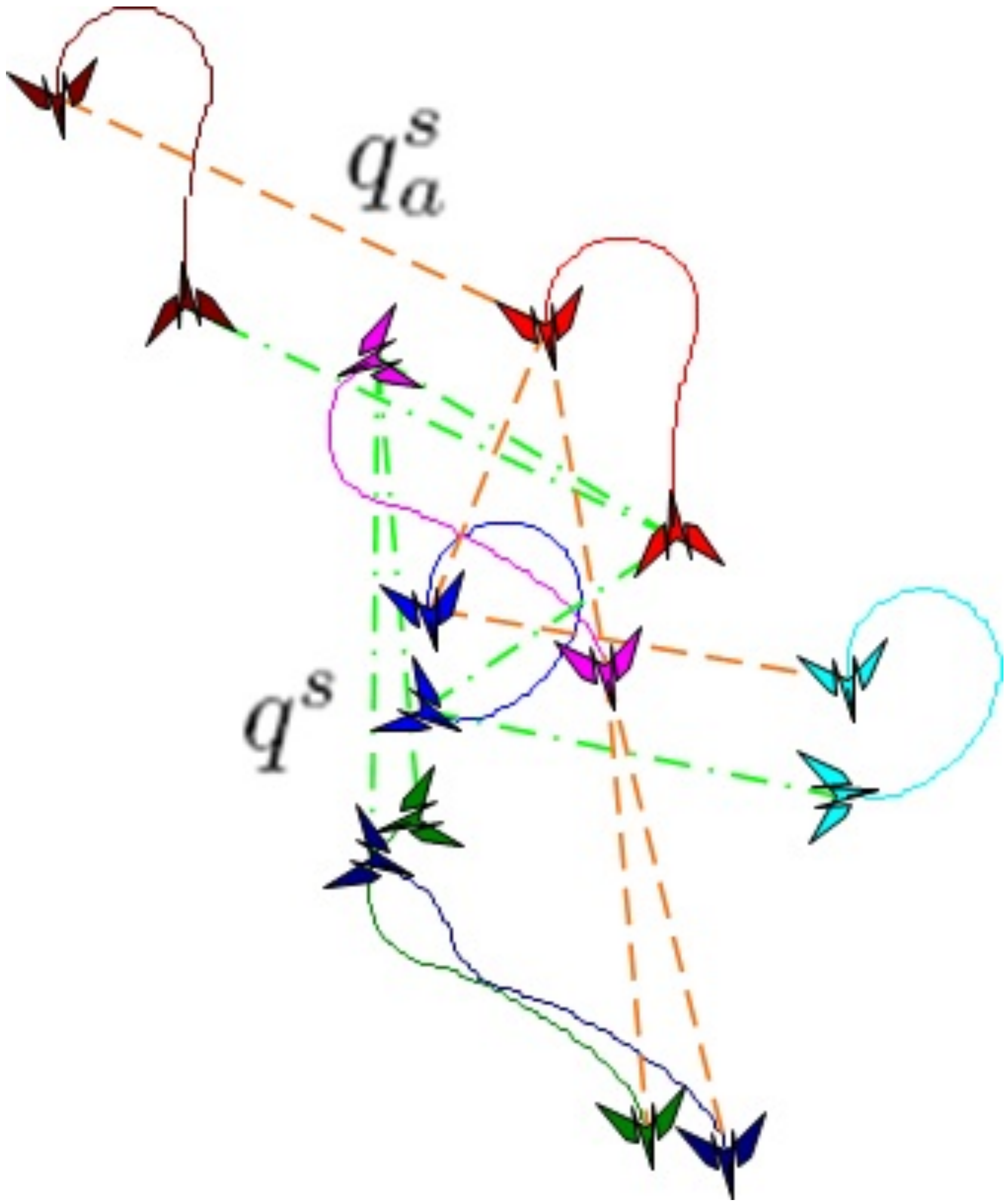
Motion Planning for Tree Formations



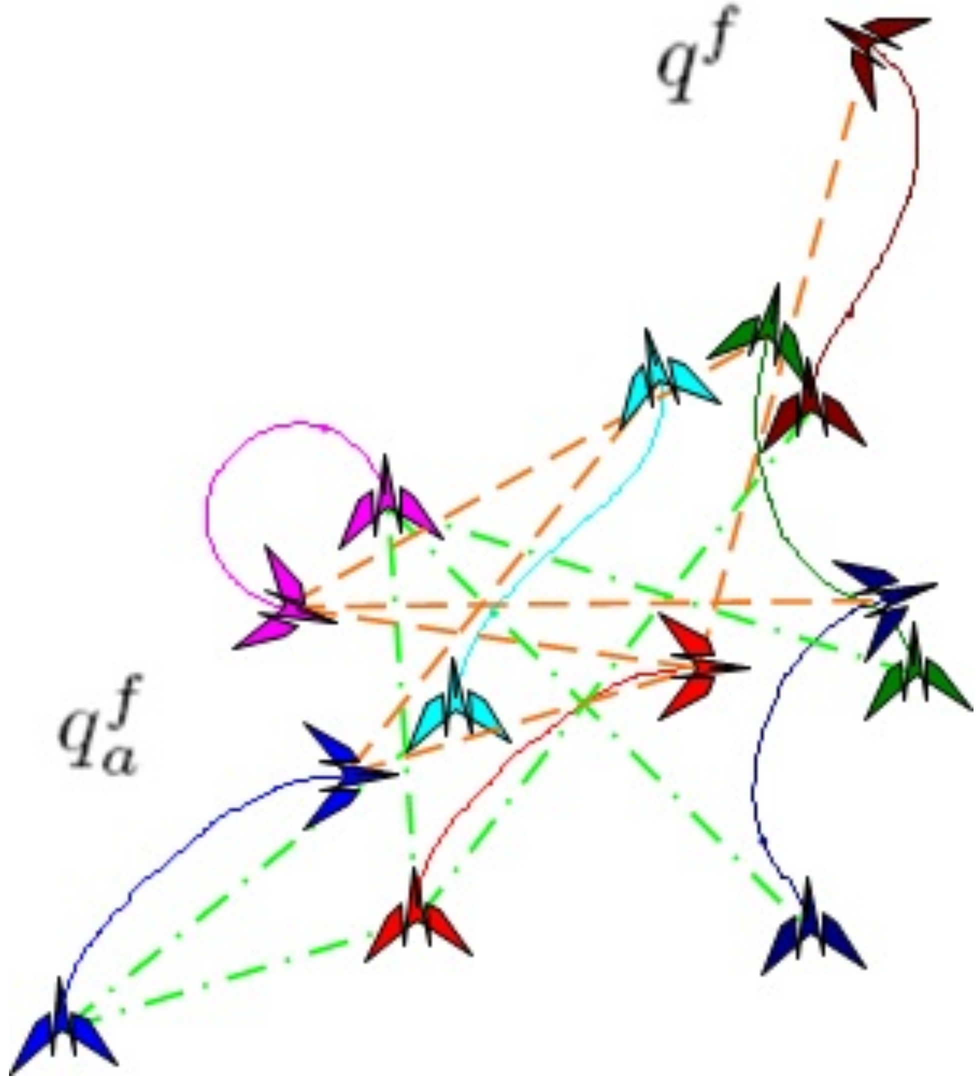


All a type!

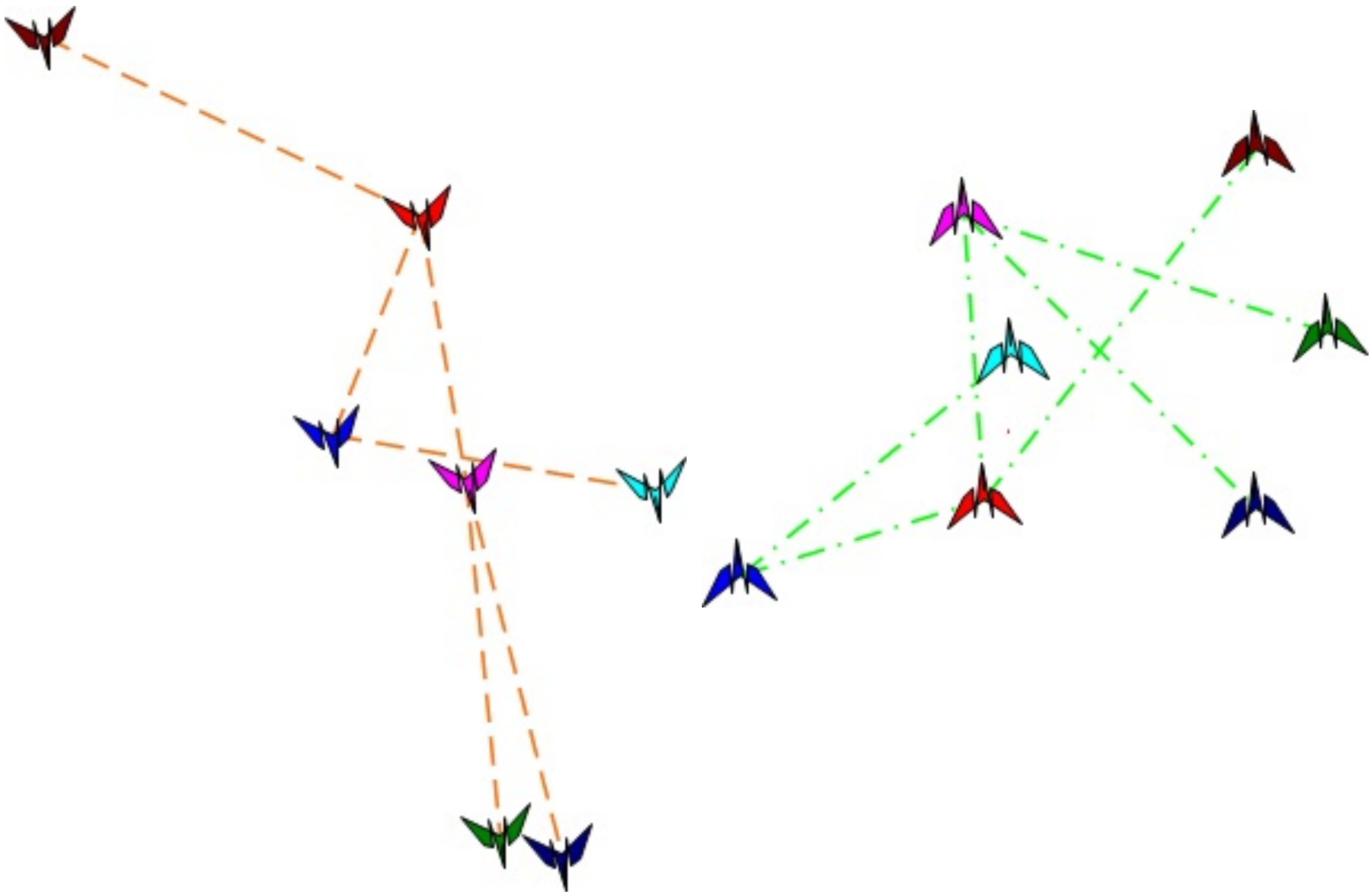
Motion Planning for Tree Formations



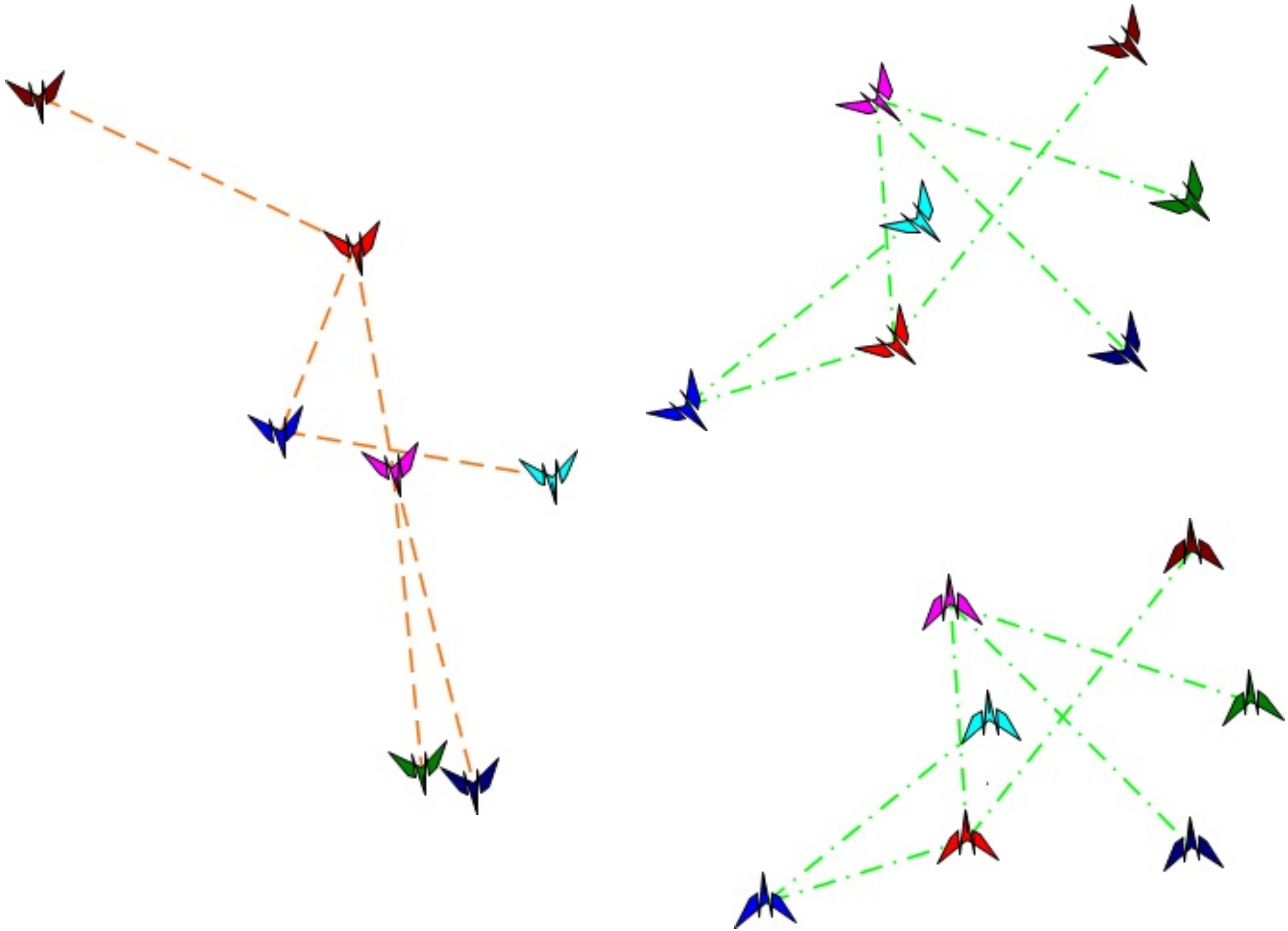
Motion Planning for Tree Formations



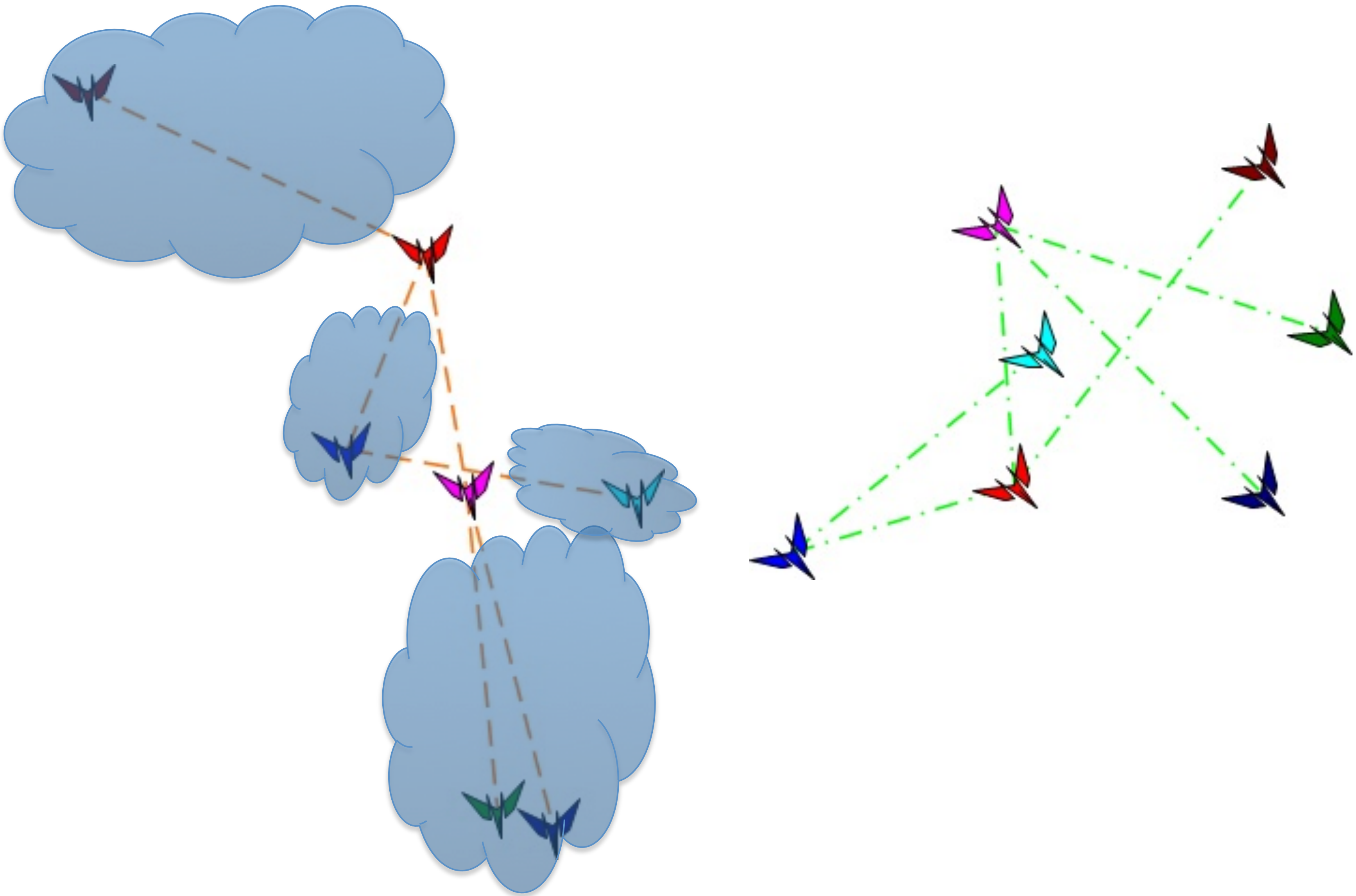
Motion Planning for Tree Formations



Motion Planning for Tree Formations

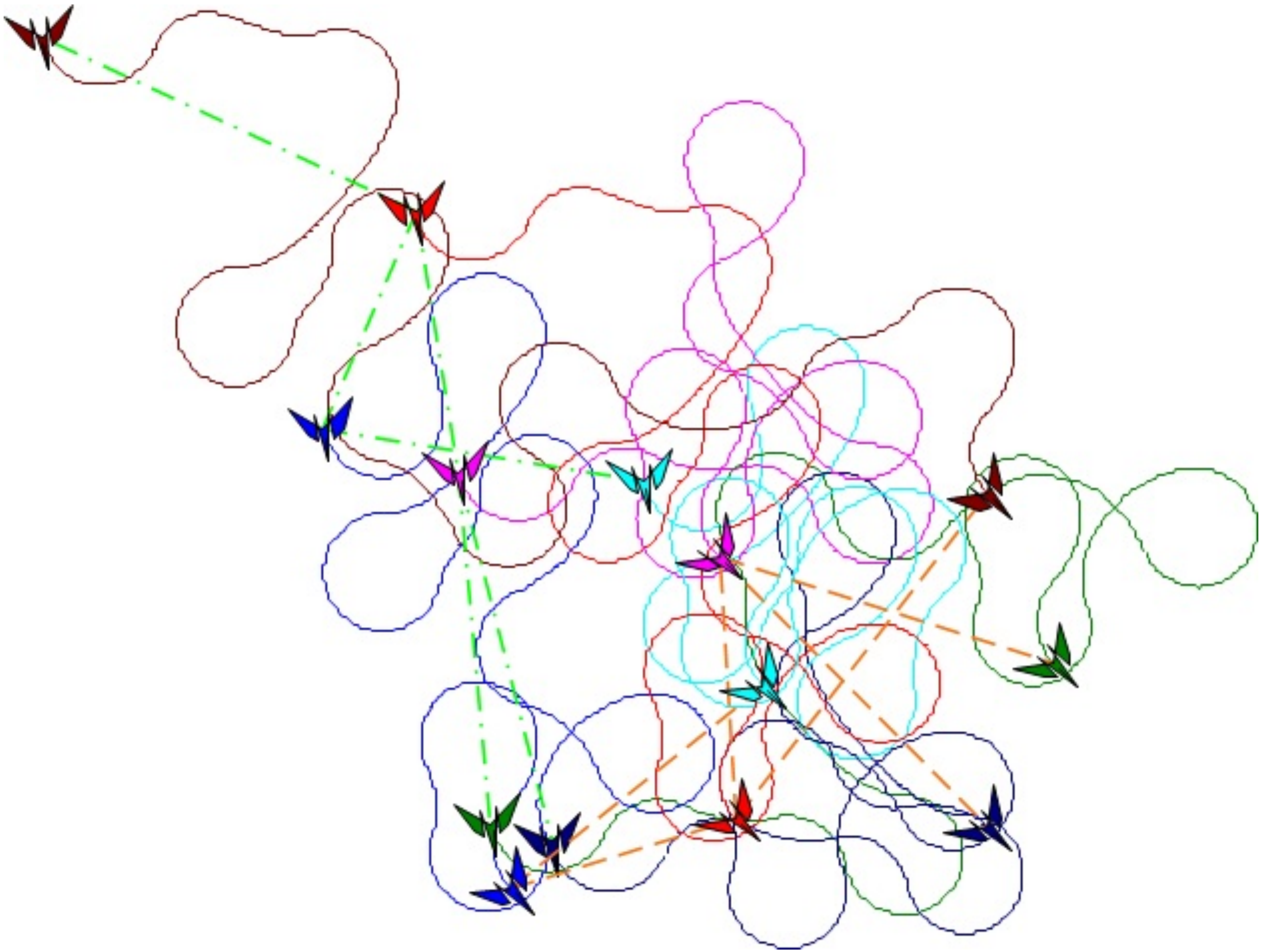


Motion Planning for Tree Formations

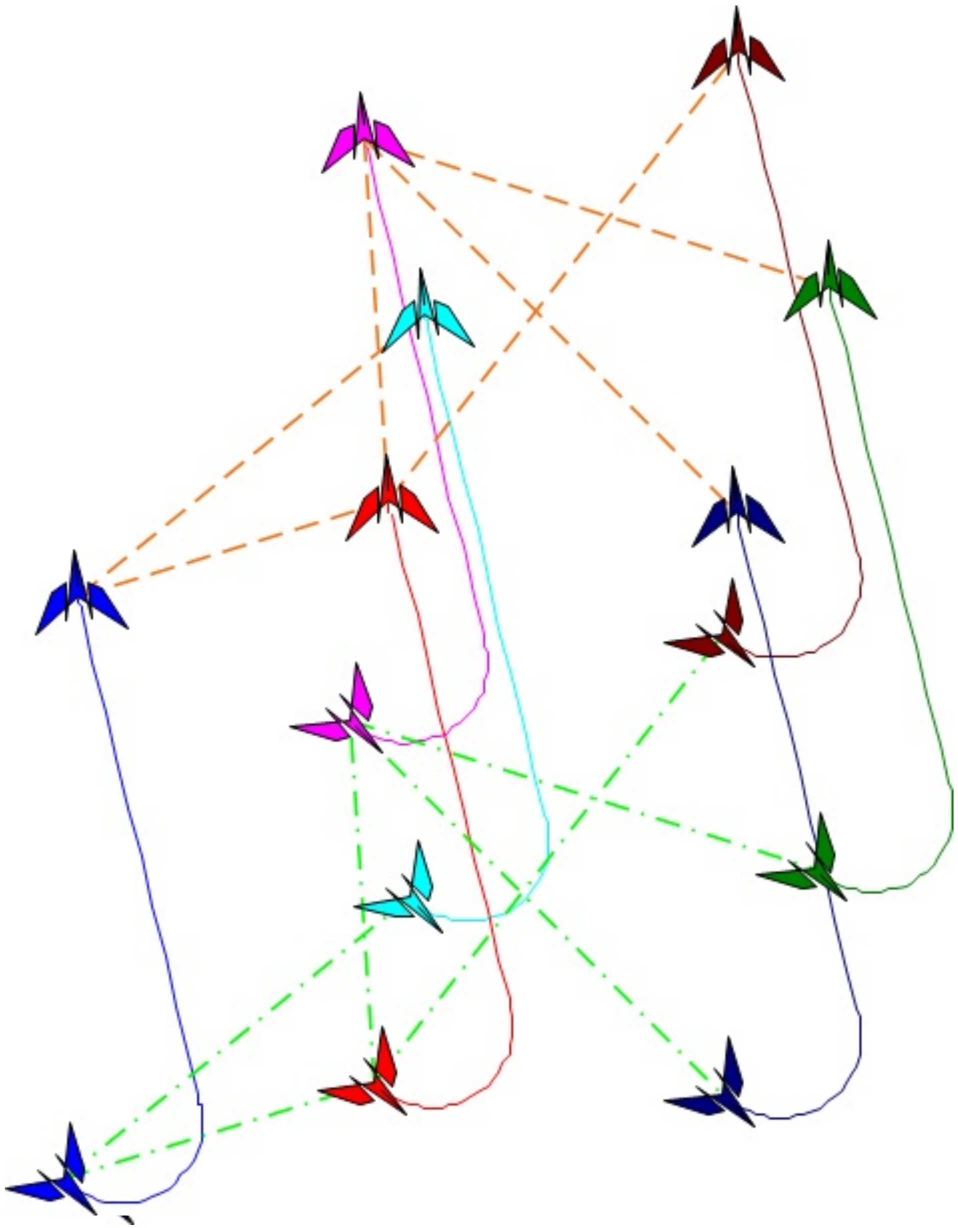


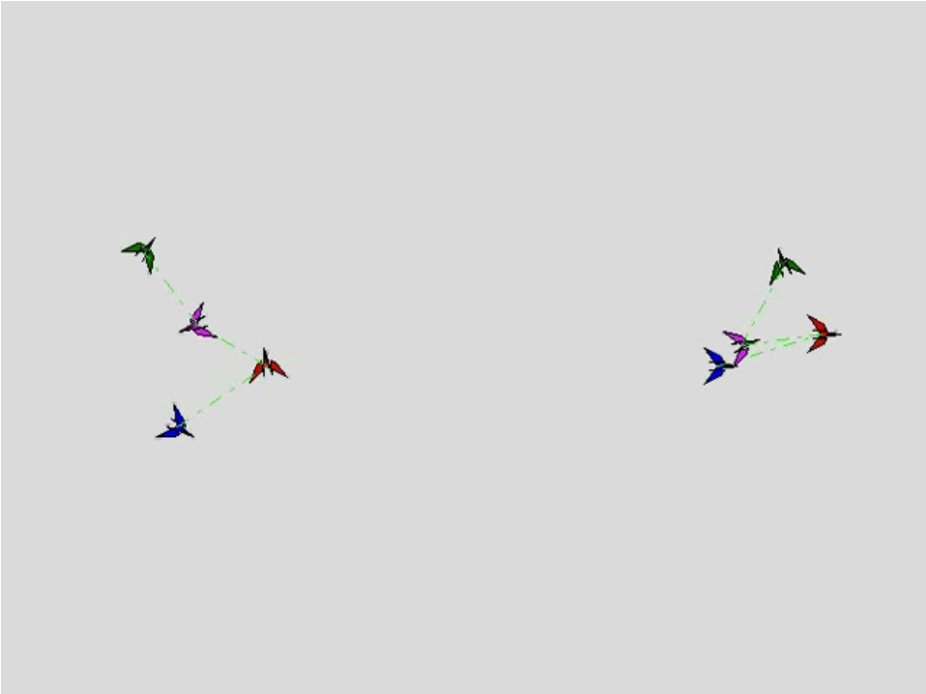
All pairs but one move as **a** type – Adjusting the angle pair by pair

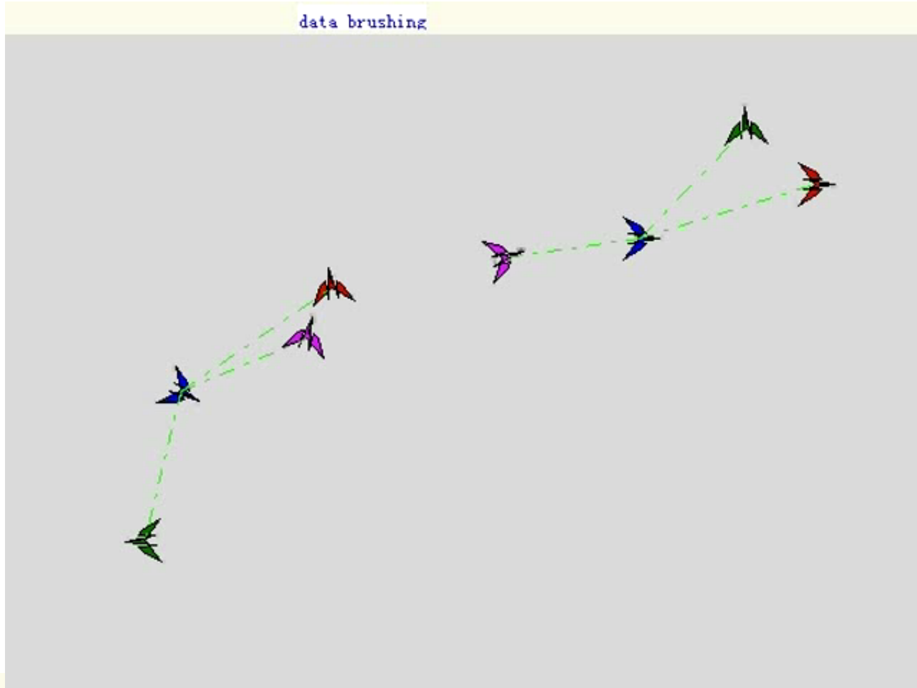
Motion Planning for Tree Formations

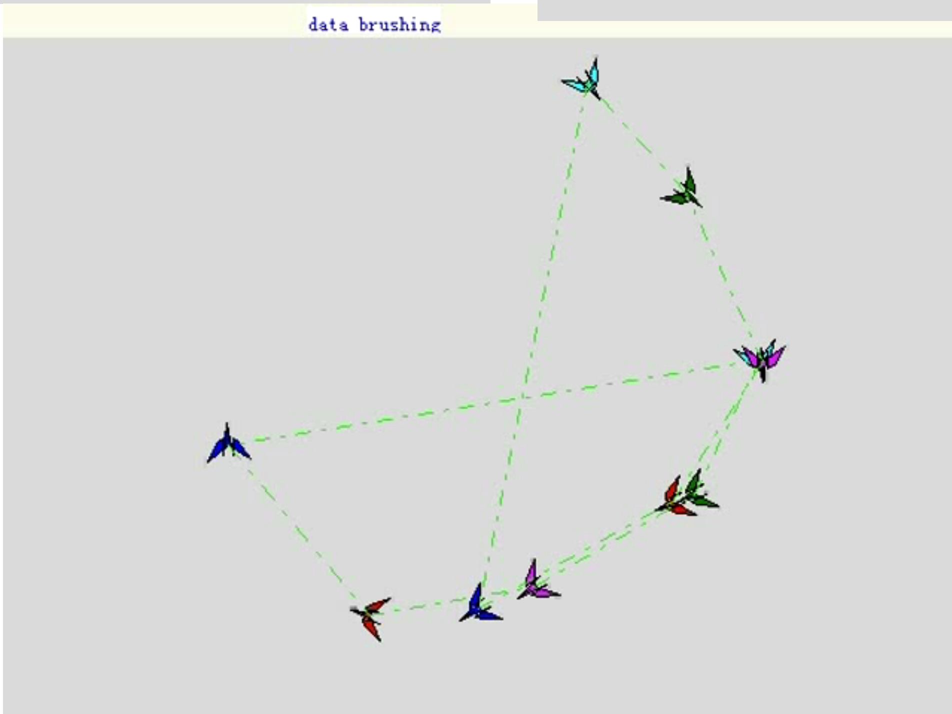


Motion Planning for Tree Formations

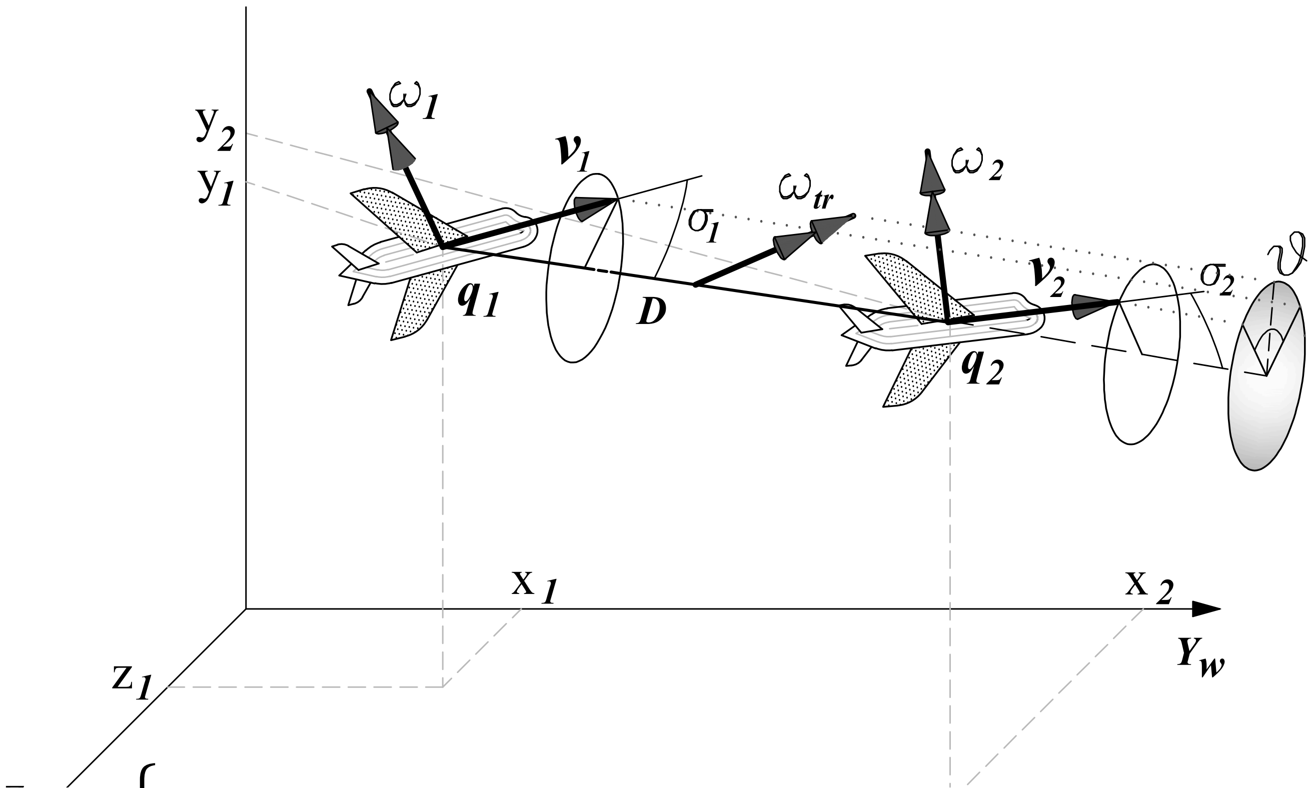





data brushing


data brushing

3D-Dubins



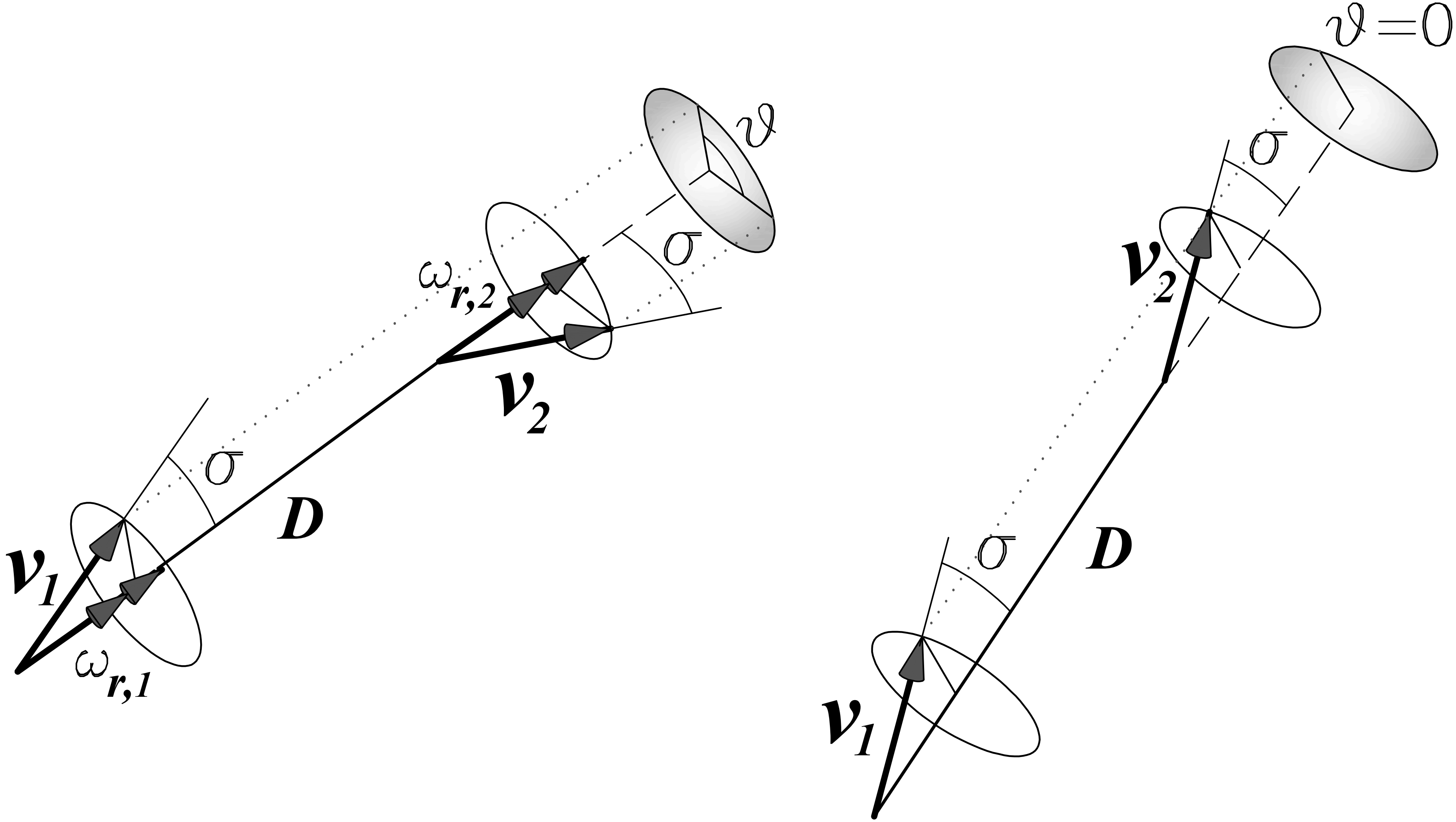
$$
\begin{cases}
\dot{q}_1 = v_1 \\
\dot{q}_2 = v_2 \\
\dot{v}_1 = v_1 \times \omega_1 \\
\dot{v}_2 = v_2 \times \omega_2
\end{cases}
$$

$$D = q_2 - q_1$$

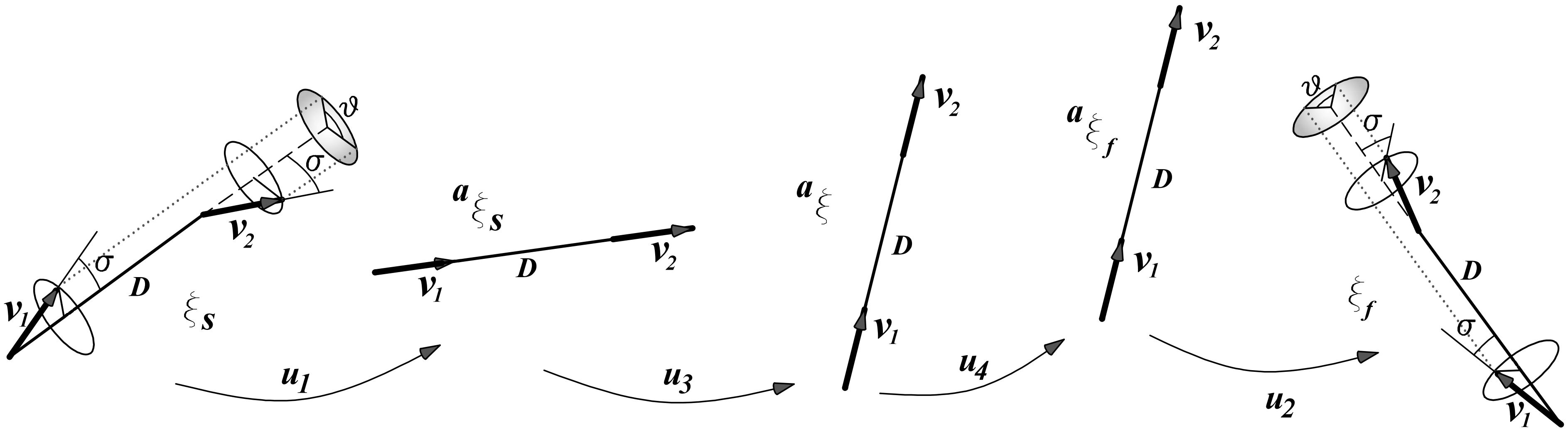
$$\dot{D} = v_2 - v_1$$

$$\dot{D} \cdot D = 0$$

3D-Dubins main maneuvers



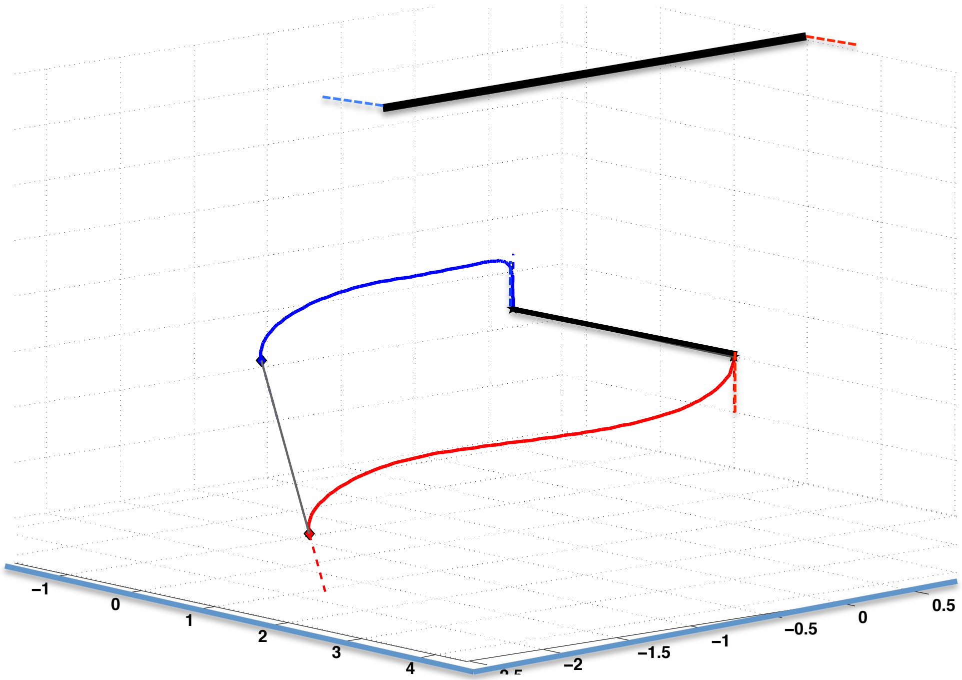
3D-Dubins motion planning



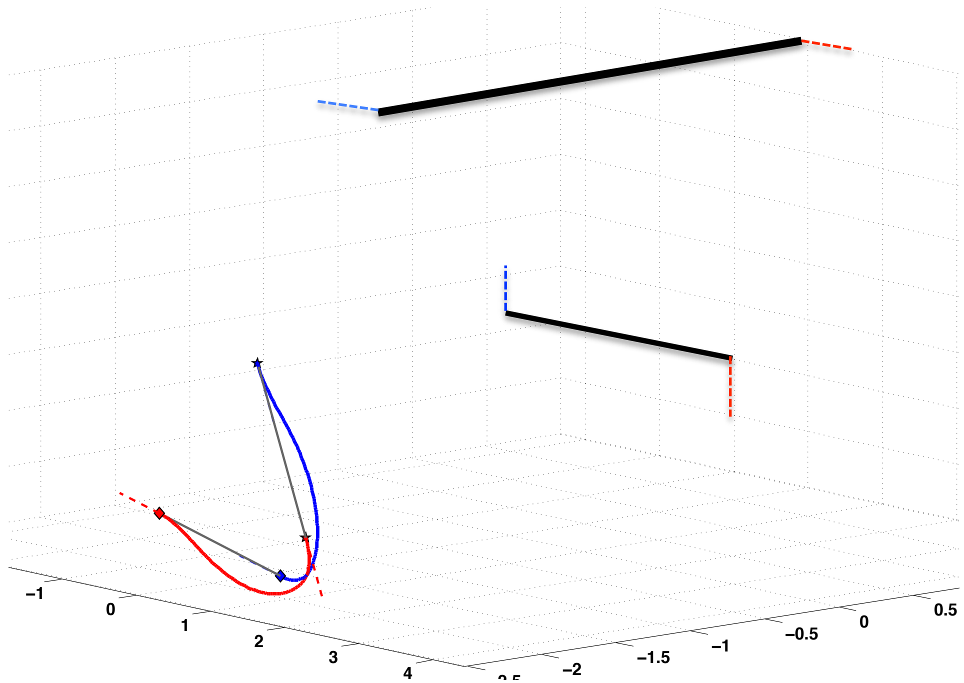
Sufficient conditions on initial and final configuration for controllability

Working on necessary conditions

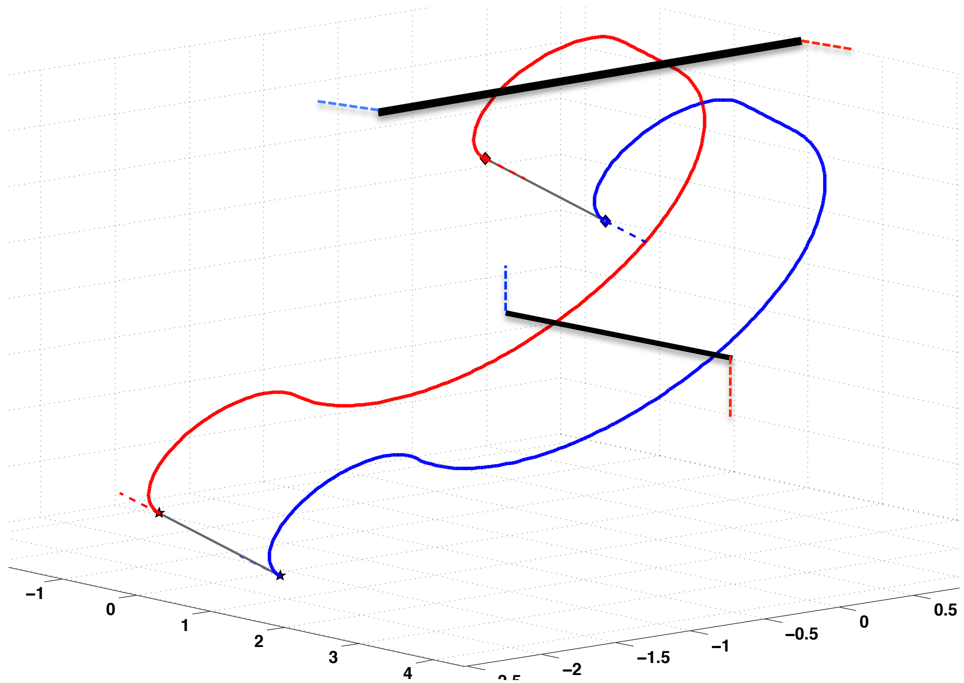
3D-Dubins motion planning



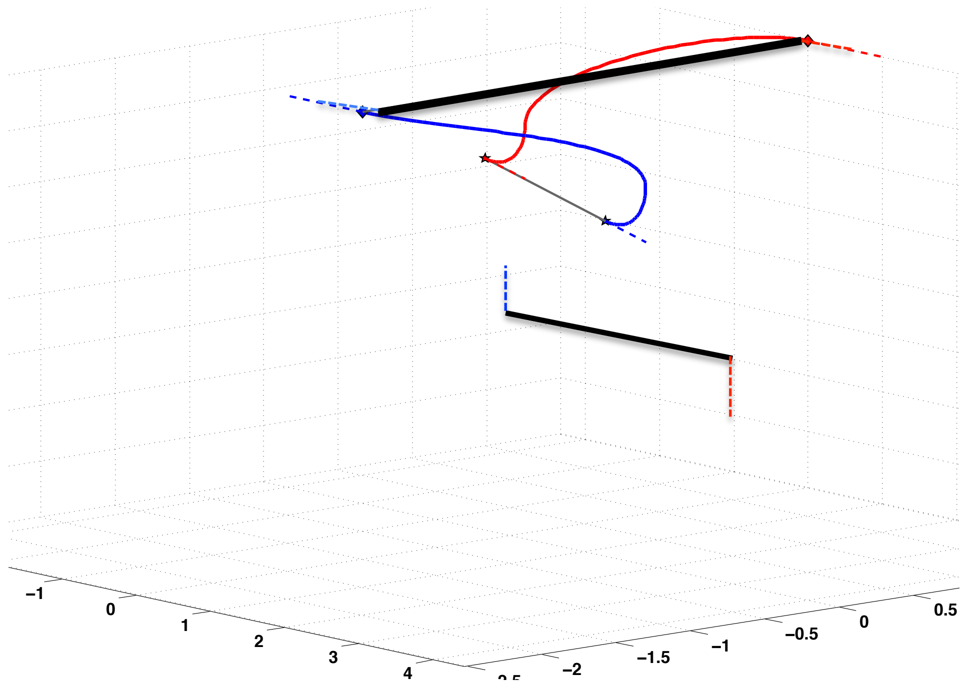
3D-Dubins motion planning



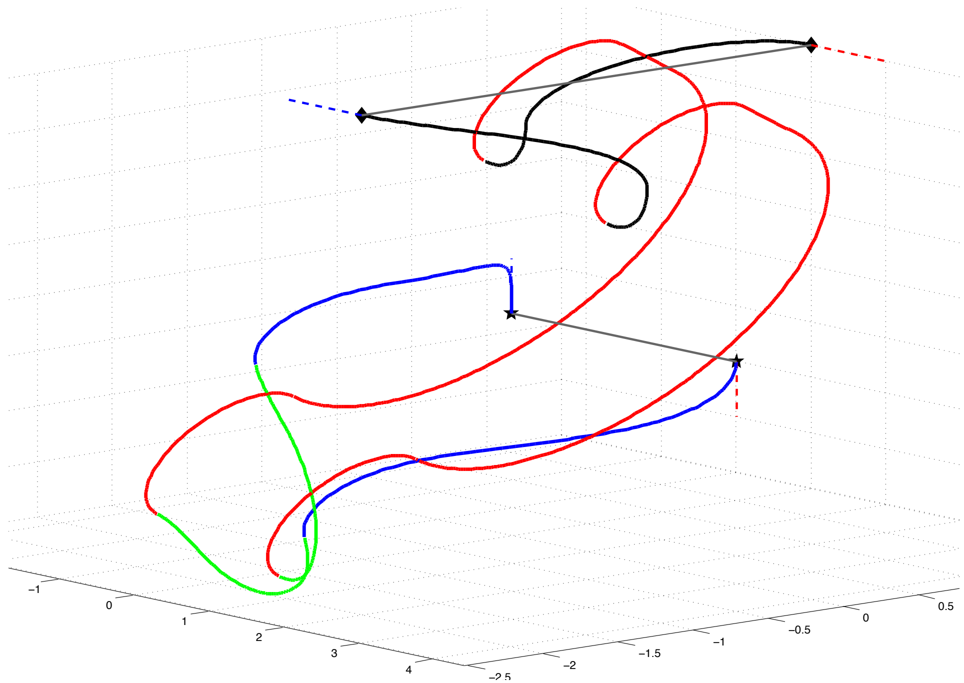
3D-Dubins motion planning



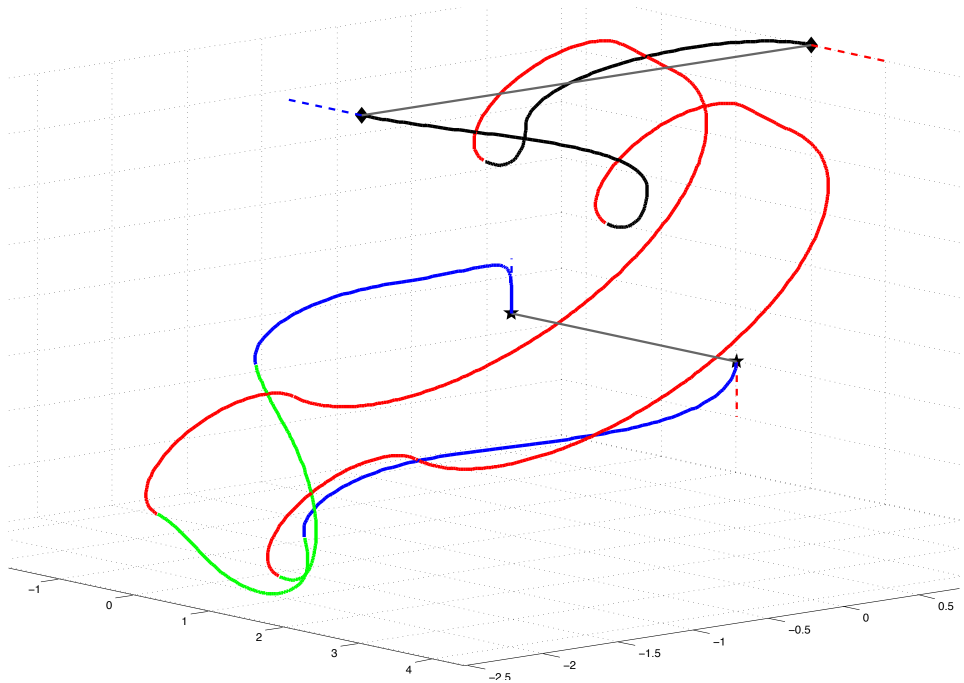
3D-Dubins motion planning



3D-Dubins motion planning



3D-Dubins motion planning



Conclusions

Solved problem: controllability and motion planning for tree formations

On going research:

controllability and motion planning for ring formations

controllability for pairs of 3D-Dubins

Far from optimal but closer than before

Motion planning can be done in a distributed way

Thank you!

Centro E. Piaggio
bioengineering and robotics research center