Grasp and Manipulation Analysis for Synergistic Underactuated Hands Under General Loading Conditions

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Abstract— **In dexterous grasping, the development of simple but practical hands with reduced number of actuators, designed to perform some manipulation tasks, is both attractive and challenging. To carefully synthesize inter- and intra-finger couplings a rigorous way to establish grasping and manipulation properties of an underactuated hand is of paramount importance. In this paper, we propose a general approach to characterize the structural properties of underactuated hands focusing on their kinematic and force analysis. A complete *kinostatic* characterization of a given grasp (pure squeeze, spurious squeeze, kinematic grasp displacements and so on) is introduced. The analysis is quasi-static but it is not limited to rigid-body motions, encompassing also essential elastic motions, statically indeterminate configurations, and pre-loaded initial conditions. The introduction of generalized compliance at contacts and in the actuation mechanism is included, as it is an essential feature of safe and dependable modern hands. Efficient algorithms to characterize the system behavior are presented and applied in two different numerical examples.**

I. INTRODUCTION

In the design of robotic hands, the capability to adapt to different tasks usually leads to complex kinematic structures with a high number of DoFs, which may increase the size, complexity and weight of the device. Robotic hand designs have attempted, over the years, to imitate the human hand in terms of dexterity and adaptation capabilities. Some remarkable example of robotic hand designs are the DLR hand II [5] and the UTAH/MIT hand [10]. One of the main issues in designing and controlling robotic hands is that a large number of motors is needed to fully actuate the degrees of freedom — but this comes at the cost of size, complexity and weight of the device.

A possible approach to reduce complexity it to cleverly cut down on the number of actuators: the outcome are underactuated hands that are simpler and more reliable than their fully actuated counterparts [11], [2], [3].

Recently, studies on human hands postural synergies [14] have inspired new researches on design and control strategies for robotic hands whose main goal is to achieve a trade-off between simplicity and versatility [4], [7]. In [7], the synergy idea has been applied to control different hand models: a simple gripper, the Barrett hand, the DLR hand, the Robonaut hand, and the human hand model. In [4], the authors proposed a robotic hand design able to match postural synergies by mechanically coupling the motion of the single joints. In [15] a synergy impedance controller was derived and implemented on the DLR Hand II.

Reducing the number of control inputs lowers the dimension of the force and motion controllability subspaces thus affecting the dexterity of the grasp. In [13] the authors investigated to what extent a hand with many DoFs can exploit postural synergies to control force and motion of the grasped object, while [8] analysed how the engaged synergy affect the *quality* of a grasp, in terms of suitably defined cost functions.

In this paper, we proceed in the analysis of underactuated hands introducing a quasi-static representation of the hand grasp that includes compliance in both contacts and in the actuation mechanism. The main aspect here is that in underactuated hands often the force problem cannot be univocally solved within the rigid-body framework, because of static indeterminacy [13], [8]. To this end, actuation system and contact compliance must be brought into play, as discussed in [1]. Then, we complement previously developed models [13], [8] (which take into account also the compliance in the underactuated synergy space), by including the effects of Jacobian, grasp matrix and synergy matrix variations. These terms cannot be neglected [6] if the system is very compliant and/or the analysis starts from a pre-loaded condition.

The paper is organized as follows: Section II presents the notation and the preliminaries of modelling quasi-static grasps. Section III presents a classification of some grasping actions. Section IV shows two procedures to calculate the perturbed configuration of the system as function of the input values. To conclude, Section V presents numerical results related to the analysis of two simple grippers showing the validity of the approach and highlighting the effect of underactuation in grasping and manipulation performances.

II. PRELIMINARIES

With reference to Fig. 1, we model a hand as a collection of serial robot manipulating an object. An inertial frame $\{A\}$ is attached to the palm.

On the i^{th} of the n contact points we place a frame $\{C_i^h\}$ attached to the link, and a frame $\{C_i^o\}$ fixed with the object.

Denoting with $g_{ac_i^h} \in SE(3)$ the posture of $\{C_i^h\}$ with respect to $\{A\}$, by means of the POE parametrization we

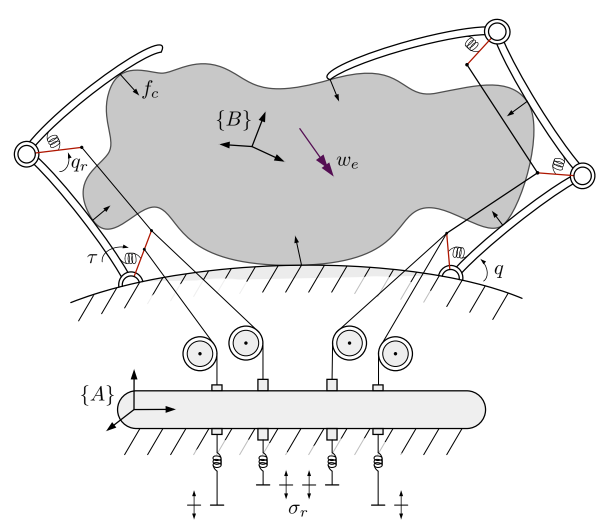

Fig. 1: Compliant grasp by an underactuated robotic hand.

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Notation	Definition
$\{A\}$	palm (inertial) frame
$\{C_i^h\}$	frame attached to the hand for the i^{th} contact point
$\{C_i^o\}$	frame attached to the object for the i^{th} contact point
$\{B\}$	barycentric object frame
m	number of hand joints
$q \in \mathbb{R}^m$	joint parameters
$q_r \in \mathbb{R}^m$	reference joint parameters
$\tau \in \mathbb{R}^m$	joint torques
s	number of postural synergies
$\sigma \in \mathbb{R}^s$	synergistic displacements
$\sigma_r \in \mathbb{R}^s$	synergistic reference displacements
$\eta \in \mathbb{R}^s$	synergistic generalized forces
n	number of contact points
c	number of contact constraints
$f_{c^h}^{c^o} \in \mathbb{R}^c$	contact force/torque vector exerted by the hand on the object
$u \in \mathbb{R}^d$	pose of the object frame; $d = 6$ (3D), $d = 3$ (2D)
$\xi_{xy}^z \in \mathbb{R}^6$	twist of frame $\{Y\}$ with respect to $\{X\}$ in components $\{Z\}$
$v_{xy}^z \in \mathbb{R}^c$	velocity of contact points in the constrained directions
$w_x^y \in \mathbb{R}^d$	wrench exerted from $\{X\}$ to $\{Y\}$ with components and moments relative to $\{Y\}$ frame
${}^aJ \in \mathbb{R}^{c\times m}$	hand Jacobian matrix in inertial frame
${}^{c^o}J \in \mathbb{R}^{c\times m}$	hand Jacobian matrix in contact frame
$S \in \mathbb{R}^{m\times s}$	synergy matrix
${}^bG \in \mathbb{R}^{d\times c}$	grasp matrix in body frame
A	*Global Grasp Matrix*, it is the coefficient matrix of the *Global Grasp Equation* (20)
Γ	base for the nullspace of the matrix A
δx	variation of variable x
$\bar{x}$	value of variable x in the reference configuration
$\sharp x$	dimensions of vector x

TABLE I: Notation for grasp analysis.

can write

$$g_{ac_i^h} = \left[\prod_{k=1}^{m_i} e^{\hat{\xi}_k q_k}\right] g_{ac_i^h}(0) \tag{1}$$

where $\xi_k \in se(3)$ are the basis elements ($\hat{\xi}_k \in \mathbb{R}^{4\times 4}$ is its homogeneous form), the q_k's are the exponential coordinates, and $g_{ac_i^h}(0)$ is the initial configuration (see [12] for more details). Other definitions are summarized in Table I.

A. Object Equations

1) Equilibrium Equation: in order to describe the actions on the object in a local frame, a right-invariant formulation is adopted. With the notation in Table I, the *equilibrium equation* for the object can be written as

$$w_e^b + {}^bG f_{c^h}^{c^o} = 0. \tag{2}$$

where ${}^bG \in \mathbb{R}^{d\times c}$ is the *grasp matrix* in body frame. By employing a right-invariant description of the system, the grasp matrix is constant. Therefore, the object equilibrium equation for a perturbed configuration is simply

$$\delta w_e^b + {}^bG \delta f_{c^h}^{c^o} = 0. \tag{3}$$

2) Congruence Equation: defining $v_{ab}^{c^o} \in \mathbb{R}^c$ as a vector collecting the velocity of all contact points in the constrained directions, by duality the relationship between the object twist and the contact point velocities is

$$v_{ab}^{c^o} = {}^bG^T \xi_{ab}^b. \tag{4}$$

Since the rotational part of the twist ξ_{ab}^b is not an exact differential, we introduce a local parametrization $u \in \mathbb{R}^d$. In this way, by a suitably defined Jacobian matrix $T(u) \in \mathbb{R}^{6\times 6}$, the twist is expressed as a function of the time derivatives of (true) coordinates u as follows

$$\xi_{ab}^b = {}^bT(u)\dot{u}. \tag{5}$$

Defining the displacements of the object contact frame as $\delta C_{ab}^{c^o} = \xi_{ab}^{c^o} dt$, by virtue of equations (4) and (5), we finally get

$$\delta C_{ab}^{c^o} = {}^b\tilde{G}^T(u)\delta u \tag{6}$$

where ${}^b\tilde{G}^T(u) = {}^bG^T\,{}^bT(u)$.

B. Hand Equations

1) Congruence Equation: letting the vector $\xi_{ac_i^h}^a \in \mathbb{R}^6$ represent the twist of $\{C_i^h\}$ with respect to $\{A\}$, with components expressed in $\{A\}$, it is possible to write

$$\xi_{ac_i^h}^a = {}^aJ_i(q_i)\dot{q}_i, \tag{7}$$

where ${}^aJ_i(q_i) \in \mathbb{R}^{6\times m_i}$ is the *spatial Jacobian* [12] for the i^{th} contact point. Again, adopting a right-invariant description of the problem we need to properly map the expressions to the object frame $\{C_i^o\}$. By employing the adjoint operator Ad we can write

$$\xi_{ac_i^h}^{c_i^o} = \mathrm{Ad}_{g_{c_i^o a}(u)}\, \xi_{ac_i^h}^a = \mathrm{Ad}_{g_{c_i^o a}(u)}\, {}^aJ_i(q_i)\dot{q}_i. \tag{8}$$

To select only the terms that are acting on the local spring we pre-multiply the twist for the (selection) matrix B^T, thus getting

$$v_{ac_i^h}^{c_i^o} = B^T \mathrm{Ad}_{g_{c_i^o a}(u)} {}^aJ_i(q_i)\dot{q}_i = {}^{c_i^o}J_i(q_i,u)\dot{q}_i. \tag{9}$$

where ${}^{c_i^o}J_i(q_i,u) = B^T \mathrm{Ad}_{g_{c_i^o a}(u)} {}^aJ_i(q_i) \in \mathbb{R}^{c_i\times m_i}$ is the Jacobian matrix for the i^{th} contact point on the hand referred to the frame attached to the object. By collecting all joint parameters of the hand in the vector $q \in \mathbb{R}^m$, from equation (9) we can write

$$v_{ac^h}^{c^o} = {}^{c^o}J(q,u)\dot{q}, \tag{10}$$

where ${}^{c^o}J(q,u)$ is the *hand Jacobian* referred to object contact points. Multiplying by dt equation (10) we can find a relationship for the displacement of the frames as

$$\delta C_{ac^h}^{c^o} = {}^{c^o}J(q,u)\delta q. \tag{11}$$

2) Equilibrium Equation: to find a relationship between joint torques and wrench exerted by the finger on contact points, we rely on the well known the kineto-static duality. By taking (10) into account, the sought map is given by the equation

$$\tau = {}^{c^o}J^T(q,u) f_{c^h}^{c^o}. \tag{12}$$

To find a relationship valid for "small" perturbations of the involved quantities, we differentiate equation (12). Introducing $p := (q, u, f_{c^h}^{c^o})$ and $\bar{x}$ as indicative of the value of x in the reference configuration, the varied equilibrium equation is (13). Therefore, the joint and object displacements and contact force variations are constrained by the following equation

$$\delta\tau = \bar{Q}\delta q + \bar{U}\delta u + \bar{J}^T \delta f_{c^h}^{c^o} \tag{14}$$

where symbols $\bar{Q}$, $\bar{U}$ and $\bar{J}^T$ can be recovered from (13).

3) Elastic Joint Model: for the i^{th} joint we introduce the *joint stiffness* $k_{q_i} \in \mathbb{R}$ that relates the joint torque with the difference between a reference configuration q_{r_i} and the real one. Incidentally, k_{q_i} can be interpreted as the steady-state gain of a position controller. Defining $K_q = \mathrm{diag}(k_{q_1}, \ldots, k_{q_m}) \in \mathbb{R}^{m\times m}$ with all the joint stiffness values on its diagonal, and introducing the vector $\delta q_r = [\delta q_{r_1}, \ldots, \delta q_{r_m}]^T \in \mathbb{R}^m$ collecting the joint reference variations, we can describe the variation of the joint torques as

$$\delta\tau = K_q(\delta q_r - \delta q). \tag{15}$$

$$\delta\tau = \left.\frac{\partial\, {}^{c^o}J^T(p) f_{c^h}^{c^o}}{\partial q}\right]_{\bar{p}} \delta q + \left.\frac{\partial\, {}^{c^o}J^T(p) f_{c^h}^{c^o}}{\partial u}\right]_{\bar{p}} \delta u + \left.\frac{\partial\, {}^{c^o}J^T(p) f_{c^h}^{c^o}}{\partial f}\right]_{\bar{p}} \delta f_{c^h}^{c^o} \tag{13}$$

4) Introducing Synergies: for the sake of extending the analysis to under-actuated hands, we define a vector of *postural synergies* $\sigma \in \mathbb{R}^s$, with $s \leq m$. Irrespective of the general (possibly nonlinear) relationship between joint reference q_r and synergies σ, under mild technical conditions, the relationship between joint reference and synergistic variable variations can always be expressed as

$$\delta q_r = S(\sigma)\delta\sigma. \tag{16}$$

Again, by virtue of the kineto-static duality, the following relationship holds

$$\delta\eta = S^T(\sigma)\delta\tau + \Sigma(\sigma,\tau)\delta\sigma, \tag{17}$$

where the η's are generalized forces at the level of synergies and $\Sigma(\sigma,\tau)=\frac{\partial S^T(\sigma)\tau}{\partial\sigma}\in\mathbb{R}^{s\times s}$.

As already done for the joints, we introduce an elastic model for the actuation. We can define the *synergistic stiffness matrix* K_σ=diag$(k_{\sigma_1},\ldots,k_{\sigma_m})\in\mathbb{R}^{s\times s}$ as a matrix collecting all *synergistic stiffness* values $k_{\sigma_i}\in\mathbb{R}$. Collecting all the *synergistic reference variations* in the vector $\delta\sigma_r = [\delta\sigma_{r_1},\ldots,\delta\sigma_{r_s}] \in \mathbb{R}^s$, the variation of actuation forces is described by the equation

$$\delta\eta = K_\sigma(\delta\sigma_r - \delta\sigma). \tag{18}$$

C. Hand/Object Interaction Model

In the general case the force distribution problem can be statically indeterminate. To properly tackle this problem we introduce a set of virtual springs at the interface of the contacting frames $\{C_i^h\}$ and $\{C_i^o\}$ on the hand and the object side, respectively. For the i^{th} contact point we introduce a *stiffness matrix* $K_{c_i}\in\mathbb{R}^{c_i\times c_i}$ containing the characteristics of the virtual springs. Defining $K_c = \text{blkdiag}(K_{c_1},\ldots,K_{c_n})$ as a matrix collecting all the contact stiffness, the constitutive equations for all the contacts become

$$\delta f_{c^h}^{c^o} = K_c(\delta C_{ac^h}^{c^o} - \delta C_{ab}^{c^o}). \tag{19}$$

D. The Global Grasp Equation

Casting *equilibrium*, *constitutive* and *congruence* equations (3), (6), (11), (14), (15), (16), (17), (18) and (19) in matrix form as in (20), a *linear* and *homogeneous* system can be formed which describes the properties of the whole manipulation system in the vicinity of an equilibrium. For brevity, we denote with $\sharp x$ the dimension of a vector x. Moreover, $I_{\sharp x}$ denotes the identity matrix in $\mathbb{R}^{\sharp x\times\sharp x}$.

Equation (20) appears in the form $A\delta y = 0$, where $A \in \mathbb{R}^{r_a\times c_a}$ is the *global grasp matrix*, containing all the coefficients of the system, and $\delta y \in \mathbb{R}^{c_a}$, incorporating both generalized system forces and configurations, is defined as the *augmented configuration*. The fulfilment of the grasp equation (20) is the condition for the hand/object system to be in equilibrium.

It is easy to find that the dimensions r_a and c_a correspond to

$$\begin{array}{l} r_a = \sharp w + 3\sharp\tau + 3\sharp f_c + 2\sharp\sigma \\ c_a = 2\sharp w + 3\sharp\tau + 3\sharp f_c + 3\sharp\sigma \end{array} \tag{21}$$

Let us introduce $\Gamma \in \mathbb{R}^{r_\gamma\times c_\gamma}$, with $r_\gamma=c_a$ and $c_\gamma=c_a-$rank(A), as a basis for the nullspace of the coefficient matrix A, thus such that $\mathcal{R}(\Gamma) = \mathcal{N}(A)$. In most cases of practical relevance rank$(A) = r_a$ and we assume it in the rest of the paper. Exceptions are analytically possible but they refer to pathological physical configurations of marginal importance: their discussion is omitted here for brevity. As a consequence of this assumption, the number of columns of Γ is equal to $c_\gamma=c_a-r_a=\sharp w+\sharp\sigma$. Therefore, the solution of the system can be parametrized by a free-variable vector of dimension $\sharp w + \sharp\sigma$.

Naturally, the matrix Γ does not have a unique expression. However it is possible to find a systematic way to elicit relevant grasp properties by seeking some predefined sparsity patterns in Γ. These patterns are associated to subspaces of particular interest that are described and classified in the next section.

III. GRASP CLASSIFICATION

A. Internal System Perturbations

A perturbation δy of the augmented configuration is defined as *internal* if it does not involve any variation of the external wrench acting on the object, that is $\delta w_e^b = 0$. Taking into account equation (3), this is equivalent to impose that $\delta f_{c^h}^{c^o}\in\mathcal{N}({}^bG)$, and therefore to the condition of *internal force variation* [1].

1) Pure Squeeze: inside the subspace of the internal system perturbations, we can look for the *pure squeeze* of the grasped object. This is defined as a perturbation δy characterized by a variation of contact forces that do not cause any object displacement. Synthetically, a pure squeeze occurs when a solution exists of the following form

$$\begin{cases} \delta w_e^b = 0 \\ \delta f_{c^h}^{c^o} \neq 0 \\ \delta u = 0. \end{cases} \tag{22}$$

2) Spurious Squeeze: a squeeze action associated to variation of (internal) contact forces that cause a displacement of the object is defined as *spurious squeeze*. It occurs when it is possible to find a solution of the form

$$\begin{cases} \delta w_e^b = 0 \\ \delta f_{c^h}^{c^o} \neq 0 \\ \delta u \neq 0. \end{cases} \tag{23}$$

3) Kinematic Grasp Displacements: still inside the subspace of internal system perturbations, we try to identify those perturbations δy that do not involve any violation of the (rigid) kinematic contact constraints. Equivalently, these δy do not cause variations of the elastic potential energy stored in the contact springs with respect to the reference (initial) configuration. To this aim, we look for a particular system perturbation that implies a variation in the object configuration but do not involve a variation of the contact forces. Synthetically, we look for those solutions of the form

$$\begin{cases} \delta w_e^b = 0 \\ \delta f_{c^h}^{c^o} = 0 \\ \delta u \neq 0. \end{cases} \tag{24}$$

It is worth observing that, if we regard the displacements of the extremities of the contact springs at the fingertips as descriptive of elastic displacements of a deformable grasped object, the application of a kinematic grasp implies a null variation of the object shape.In this reading, the definition of *rigid object displacement* can be recovered.

B. External System Perturbations

A perturbation δy of the global configuration for the system is defined as *external* if it involves a variation of the external wrench acting on the object, that is $\delta w_e^b \neq 0$.

$$\begin{bmatrix} I_{\sharp w} & 0 & {}^bG & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{\sharp\tau} & -{}^{c^o}\bar{J}^T & 0 & -\bar{Q} & 0 & -\bar{U} & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{\sharp f_c} & 0 & 0 & 0 & 0 & 0 & 0 & -K_c & K_c \\ 0 & I_{\sharp\tau} & 0 & 0 & K_q & -K_q & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -{}^{c^o}\bar{J} & 0 & 0 & 0 & 0 & I_{\sharp f_c} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -{}^b\tilde{G}^T & 0 & 0 & 0 & I_{\sharp f_c} \\ 0 & 0 & 0 & I_{\sharp\eta} & 0 & 0 & 0 & K_\sigma & -K_\sigma & 0 & 0 \\ 0 & -S^T & 0 & I_{\sharp\eta} & 0 & 0 & 0 & -\Sigma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{\sharp q} & 0 & -S & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta w_e^b \\ \delta\tau \\ \delta f_{c^h}^{c^o} \\ \delta\eta \\ \delta q \\ \delta q_r \\ \delta u \\ \delta\sigma \\ \delta\sigma_r \\ \delta C_{ac^h}^{c^o} \\ \delta C_{ab}^{c^o} \end{bmatrix} = 0 \tag{20}$$

1) External Structural Forces: in the subspace of external actions it is possible to look for perturbations δy that are characterized by variations of the external wrench δw_e^b and contact forces $\delta f_{c^h}^{c^o}$, but do not involve any modifications of the synergistic actions and references. If any, these are equivalently characterized as

$$\begin{cases} \delta w_e^b \neq 0 \\ \delta f_{c^h}^{c^o} \neq 0 \\ \delta\eta = 0 \\ \delta\sigma_r = 0. \end{cases} \tag{25}$$

By taking into account equation (18), conditions (25) imply also $\delta\sigma = 0$.

At this point other definitions could be given to further characterize the system properties. However such a detailed classification is outside the scope of this work.

C. Grasp Classification As a Block Decomposition Of The Nullspace Matrix

To sum up, by taking into account all the definitions above, we seek a decomposition of the nullspace of A of the form

$$\begin{bmatrix} \delta w_e^b \\ \delta\tau \\ \delta f_{c^h}^{c^o} \\ \delta\eta \\ \delta u \\ \delta q_r \\ \delta q \\ \delta\sigma \\ \delta\sigma_r \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \tilde{\Gamma}_{ws_t} & \tilde{\Gamma}_{wc_o} \\ \Gamma_{\tau i} & \Gamma_{\tau s_q} & \Gamma_{\tau k} & \Gamma_{\tau s_t} & \Gamma_{\tau c_o} \\ \Gamma_{fi} & \tilde{\Gamma}_{fs_q} & 0 & \tilde{\Gamma}_{fs_t} & \Gamma_{fc_o} \\ \Gamma_{\eta i} & \Gamma_{\eta s_q} & \Gamma_{\eta k} & 0 & \Gamma_{\eta c_o} \\ \Gamma_{ui} & 0 & \tilde{\Gamma}_{uk} & \Gamma_{us_t} & \Gamma_{uc_o} \\ \Gamma_{q_r i} & \Gamma_{q_r s_q} & \Gamma_{q_r k} & \Gamma_{q_r s_t} & \Gamma_{q_r c_o} \\ \Gamma_{qi} & \Gamma_{qs_q} & \Gamma_{qk} & \Gamma_{qs_t} & \Gamma_{qc_o} \\ \Gamma_{\sigma i} & \Gamma_{\sigma s_q} & \Gamma_{\sigma k} & 0 & \Gamma_{\sigma c_o} \\ \Gamma_{\sigma_r i} & \Gamma_{\sigma_r s_q} & \Gamma_{\sigma_r k} & 0 & \Gamma_{\sigma_r c_o} \end{bmatrix} \begin{bmatrix} x_i \\ x_{s_q} \\ x_k \\ x_{s_t} \\ x_{c_o} \end{bmatrix}, \tag{26}$$

where x is a vector of free parameters and the $\tilde{\Gamma}$'s are blocks which are full column-rank.

The classification reported here has no pretensions to be exhaustive, and is limited to define some interesting behaviours. To complete the analysis, it would be appropriate to establish suitable definitions (mainly ramifications of the proposed ones) to classify other subspaces of interest. For example, it would be possible to distinguish *active* from *passive internal system perturbation*, and discriminate further between *active* and *passive pure squeeze (internal) system perturbation*, or to look for *redundant hand motions* and *indeterminate object motions*. However, they are omitted mainly for space limitations.

Quite remarkably though, any of the subspaces which are omitted here can be defined consistently and successively sought for by specifying an associated sparsity pattern in the columns of Γ. To this end, the *reduced row echelon form* (RREF) decomposition of proper portions of the basis Γ could be extensively employed and it provide a systematic way to investigate the nullspace basis. The details of the proposed method are extensively explained in an online report [9] for space limitations.

IV. PERTURBED EQUILIBRIUM CONFIGURATIONS

A suitable decomposition of the nullspace Γ, as operated in (26), helps to unveil the structural properties of the grasp configuration under investigation. However, this is not sufficient to predict which new configuration is reached by a system as a consequence of a given input. This because the solution is, in general, a *linear combination* of the columns of matrix Γ.

In accordance with plausible grasping configurations, we will consider the variables δw and $\delta\sigma_r$ as independent in the rest of the paper. This is quite a natural choice under the hypotheses of knowing the external perturbation δw_e^b, and to control the synergy references $\delta\sigma_r$, e.g. by servo motors.[1] In the next session we introduce two different methods to calculate the value of the augmented configuration δy as a function of the input variables δw and $\delta\sigma_r$. Nevertheless the methods could be easily extended to any other case of admissible inputs.

The first method (GEROME-B) has the advantage of producing input/output equations with relevant physical sense, especially if associated to a symbolic form. On the other hand, the second method (Find-X) has the advantage of a numerically more effective application.

A. GEROME-B: a Specialized Gauss Elimination Method for Block Partitioned Matrices

Since the system (20) is linear and homogeneous, we can act on the matrix A (performing a Gauss-Jordan elimination) to obtaining a new system where each vector of variables depends only on δw_e^b and $\delta\sigma_r$. In other words, for each element δy_i of the augmented configuration δy, we are looking for a relationship in the form

$$\delta y_i = W_i\, \delta w_e^b + R_i \delta\sigma_r. \tag{27}$$

To this end, preserving the integrity of the submatrix composing matrix A, we need to extend the Gauss Elementary Row Operation Method (GEROME) to process a block partitioned matrix (GEROME-B). We can write the elementary operations as:

- exchange the i^{th} row-block with the j^{th} row-block
- multiply the i^{th} row-block by a full-rank matrix Δ,
- add the i^{th} row-block with the j^{th} row-block possibly multiplied for a suitable matrix Λ to accord the dimensions.

Each rule can be performed by pre-multiplying matrix A by a suitable full rank matrix.[2] As a consequence, GEROME-B does not alter the solution space of the starting system while performing a Gauss-Jordan elimination in matrix A.

[1] Alternatively, should we have a hand with torque controlled motors, we could use the variables $\delta\eta$ as independent inputs. Similarly, should we have an estimate of the object displacements, we could consider δu as input from the object side, and so on.

[2] Remember that if $M \in \mathbb{R}^{m\times m}$, for any matrix $A \in \mathbb{R}^{m\times n}$ it results $\mathcal{N}(MA) = \mathcal{N}(A)$ if M is full rank.

With the basic ingredient: a partitioned identity matrix I_p, such that the i^{th} block I_{p_i} has the same dimension of the number of rows the corresponding i^{th} row of matrix A, the three equivalent GEROME-B (elementary operation) matrices can be written as

$$\begin{aligned} M^1_{ij} = \text{diag}(I_{p_1}, & \ldots, I_{p_{i-1}}, I_{p_j}, I_{p_{i+1}}, \ldots \\ & \ldots, I_{p_{j-1}}, I_{p_i}, I_{p_{j+1}}, \ldots, I_{p_m}) \\ M^2_{ii}(\Delta) = \text{diag}(I_{p_1}, & \ldots, I_{p_{i-1}}, \Delta, I_{p_{i+1}}, \ldots I_{p_m}) \\ M^3_{ij}(\Lambda) = I_p \oplus \Lambda_{ij} & \end{aligned} \tag{28}$$

where the expression $I_p \oplus \Lambda_{ij}$ indicates the default partitioned identity matrix I_p with the insertion of the matrix Λ on the block on the i^{th} row and j^{th} column. Then, in order to perform the Gauss-Jordan elimination, we first define the *pivot* elements. A block of the matrix A can be a pivot if and only if it follows the general rules:

- it is the only pivot in its row and column
- it is not a coefficient of one of the input variables
- it is a full-rank square block.

To clarify things, we suppose to operate on a particular permutation $\hat{A}$ of the initial matrix A such that all the pivot elements are on the first diagonal and the blocks multiplying the input variables are in the last columns. A general block in the i^{th} row and j^{th} column is named $\hat{A}_{ij}$. It is always possible and easy to obtain this form with a suitable permutation. It can occur that a block we need to be a pivot is a zero block. It is possible to solve this situation adding another (more) appropriate row(s) to it before starting the algorithm.

Algorithm 1 GEROME-B

```
for h = 1 → m do
  Δ = Â_hh^{-1}
  Â = [M^2_hh(Δ)] ∗ Â
  for k = 1 → m do
    if h ≠ k then
      Λ = −Â_kh
      Â = [M^3_kh(Λ)] ∗ Â
    end if
  end for
end for
```

Algorithm 1 highlights the main steps to obtain the desired form of the coefficient matrix that, multiplied for the vector δy, produce all the equations in the form (27).

B. Find-X Method

The main power of GEROME-B is to produce meaningful input/output relationships when applied in symbolic form, but it is not very efficient for numerical applications. To clear up this problem we introduce the Find-X method. Starting from a numerical form of the coefficient matrix A it is easy to obtain a generic nullspace basis Γ of it, as in (26), without partitioning. We denote with Γ_w a matrix composed by the first $\sharp w$ rows of Γ (and all the columns); similarly we denote with Γ_{σ_r} the last $\sharp\sigma$ rows. It is possible to calculate vector x in (26), responsible of the column combination of Γ, as a function of the independent variables, as follows

$$x = \begin{bmatrix} \Gamma_w \\ \Gamma_{\sigma_r} \end{bmatrix}^{-1} \begin{bmatrix} \delta w^b_e \\ \delta\sigma_r \end{bmatrix}. \tag{29}$$

The perturbed configuration can be therefore recovered as

$$\delta y = \Gamma \begin{bmatrix} \Gamma_w \\ \Gamma_{\sigma_r} \end{bmatrix}^{-1} \begin{bmatrix} \delta w^b_e \\ \delta\sigma_r \end{bmatrix}. \tag{30}$$

The invertibility of the matrix $\begin{bmatrix} \Gamma^T_w & \Gamma^T_{\sigma_r} \end{bmatrix}^T$ is guaranteed under the same general assumptions that the coefficient matrix A is full row rank. In case of different inputs it is immediate to find other suitable portions of Γ to calculate vector x as presented in (29).

V. NUMERICAL RESULTS

A. Precision Grasp

Let us consider the 2D example shown in Fig. 2, where a two fingered hand is grasping a circular object of radius R by its fingertips. Each finger is composed by two links of length L. Globally, the hand has four revolute joints $[J_1, \ldots, J_4]$. Attached to the palm we fix an inertial $\{A\}$ with origin on the intersection between the axis of J_1 and the plane. On the j^{th} contact point we introduce two reference frames $\{C^h_j\}$ and $\{C^o_j\}$ fixed with the hand and the object, respectively. Attached to the object there is a frame $\{B\}$ with origin in the center of the circle. The generic displacement of $\{B\}$ with respect to $\{A\}$ is described by the vector $\delta u^T = [\delta u^T_x, \delta u^T_y, \delta u^T_\alpha]^T$, where the first two elements indicate a linear displacement, and the last one an angular displacement. In the reference configuration all the frames are aligned with $\{A\}$. The numerical results presented later are obtained with the following numerical values: $L = 1$ m, k_q=10 N/rad, k_c=10 N/m, k_s=10 N/m, the contact model is *hard finger*, and we assume a preload $\bar{f}^{c^h}_{c^o}$=$[1, 1, -1, 1]^T$ (N), as sketched in Fig. 2.

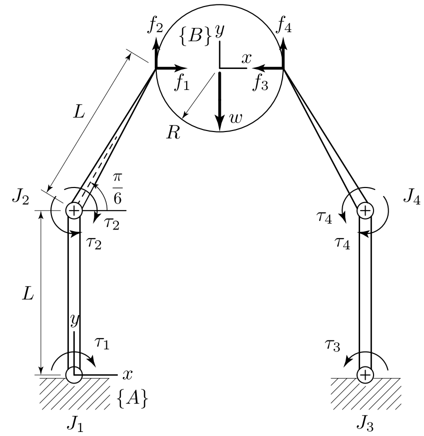

Fig. 2: Compliant grasp of a ball by a two-fingered robotic hand.

1) Perturbed Configuration for Fully Actuated Hand: the aim here is to characterize some of the structural properties of the fully actuated grasp. Therefore, we perform the decomposition of the nullspace matrix as in (26). All the above definitions hold defining a synergy matrix $S = I_{\sharp\tau}$. In this case, a basis matrix Γ for $N(A)$ has $\sharp w^b_e + \sharp\tau = 7$. Within this 7-dimensional subspace, the *pure squeeze* subspace has dimension n_{sq}=1, the subspace of *kinematic grasp displacements* has dimension $n_k = 3$: together they complete the subspace of *internal system perturbations* since n_i=0 (no *spurious squeeze* modes are present). The *external structural force* subspace has dimension n_{s_t}=0, and finally the *coordinated force and displacements* subspace has dimension n_{c_o}=3.

For the *kinematic grasp displacements*, a finite displacement of the object δu_x=0.001 in the x direction, as in Fig. 3a, is possible only if the corresponding variations in joint

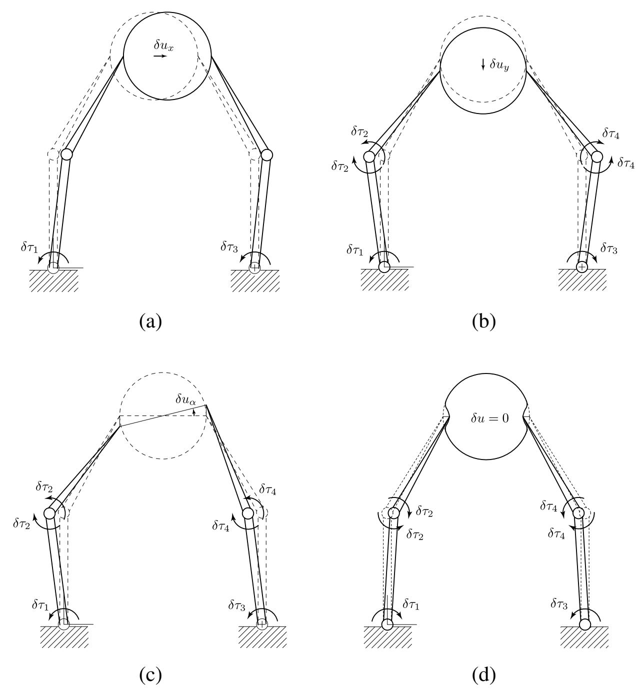

Fig. 3: Graphic representations of numerical results for the internal precision grasp variations.

torques and joint angles are

$$\begin{aligned} \delta\tau &= [0.001 \quad 0 \quad 0.001 \quad 0]^T, \\ \delta q &= [-0.001 \quad 0.001 \quad -0.001 \quad 0.001]^T, \end{aligned} \tag{31}$$

that correspond to the first column of the blocks $\Gamma_{\tau k}$ and Γ_{qk}, respectively, in equation (26).

For an object displacement δu_y=-0.001, the corresponding variations in the joint torques and joint angles are

$$\begin{aligned} \delta\tau &= [-0.0010 \quad -0.0027 \quad 0.0010 \quad 0.0027]^T, \\ \delta q &= [0.0017 \quad -0.0037 \quad -0.0017 \quad 0.0037]^T, \end{aligned} \tag{32}$$

and can be interpreted as the second column of $\Gamma_{\tau k}$ and Γ_{qk}: pictorial results are sketched in Fig. 3b. For an object rotation δu_α=0.001, the variations in the joint torques and joint angles are then

$$\begin{aligned} \delta\tau &= [0.0033 \quad 0.0040 \quad 0.0033 \quad 0.0040]^T, \\ \delta q &= [0.0017 \quad -0.0037 \quad 0.0017 \quad -0.0037]^T, \end{aligned} \tag{33}$$

corresponding to the third column of $\Gamma_{\tau k}$ and Γ_{qk}, respectively (see Fig. 3c).

Finally, the variations of joint torques and joint angles for the *squeeze* are

$$\begin{aligned} \delta\tau &= [-0.0018 \quad -0.0010 \quad 0.0018 \quad 0.0010]^T, \\ \delta q &= [-0.0001 \quad 0.0001 \quad 0.0001 \quad -0.0001]^T. \end{aligned} \tag{34}$$

This corresponds to the blocks $\Gamma_{\tau s_q}$ and $\Gamma_{q s_q}$ in (26) and it has a representation in Fig. 3d. It is important to underline the presence of the joint torque variations, caused by the terms $\bar{Q}$ and $\bar{U}$, that cannot be neglected in the presence of a contact force preload, in agreement of the results in [6].

2) A Synergy in the Precision Grasp: having sampled the properties of the fully-actuated system, we can then evaluate the effect of underactuation on the system properties by introducing just one synergy. The synergies proposed both here and later in section V-B.2, are generated (by a reverse-engineering process) after the nullspace decomposition for the fully-actuated system has been carried out, by properly combining some of the row blocks in order to synthesize a desired task. GEROME-B or Find-X are employed here to verify the results.

Introducing a single column synergy matrix as follows

$$S = \ [-0.2866 \quad 0.0034 \quad 0.2866 \quad -0.0034]^T. \tag{35}$$

we decompose the nullspace and verify what is left: one pure squeeze, $n_{s_q} = 1$, and three coordinated force and displacements, n_{c_o}=3, which together complete the basis for the nullspace since n_i=n_k=n_{s_t}=0. To elicit the nature of this synergy, we can evaluate the system response to a predictable input. To this aim, we can use the Find-X method or GEROME-B, obtaining, of course, the same results. Since the pure squeeze still exists for this system, with the above synergy it is always possible for a certain $\delta\sigma_r$ to keep the object in its starting configuration (δu=0) for a null external perturbation δw_e^b=$(0,0,0)^T$. In fact, by applying $\delta\sigma_r = 1$, we get $\delta f_c = (0.9039, 0, -0.9039, 0)$ with $\delta u = (0,0,0)^T$. This because S was defined as proportional to the (single) column $\Gamma_{q_r\, s_q}$ in (26) for the fully-actuated system.

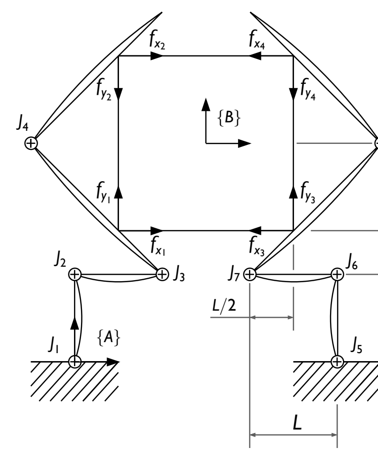

Fig. 4: Compliant grasp of a square by a two fingered spider-like hand.

B. Power Grasp

As the second test case we consider a square of side $2L$ grasped by a spider-like hand composed by two fingers and a total of 8 joints $[J_1, \ldots, J_8]$. The notation is similar to the previous example. Fig. 4 shows the starting configuration of the hand and the initial contact preload. All the force vectors have unit lengths and directions depicted. The basis Γ for the $N(A)$ is $\Gamma \in \mathbb{R}^{54\times 11}$, since A as in (20) is now $A \in \mathbb{R}^{43\times 54}$.

1) Perturbed Configuration for Fully Actuated Hand: performing the decomposition of the nullbasis Γ as in (26) it results: *squeeze* subspace of dimension n_{s_q}=5, *kinematic grasp* subspace of dimension n_k=3 (together they complete the internal subspace), a *coordinated force and displacement* subspace of dimension n_{c_o}=3. There are not other subspaces ($n_i = n_{s_t} = 0$). For the *kinematic grasp displacements*, simulation results show that it is possible to have a finite displacement of the object $\delta u_x = 0.001$, as in Fig. 5a, with no torque variations, but with a joint angle displacement of

$$\delta q = 10^{-3}\,[-1 \quad 1 \quad 0 \quad 0 \quad -1 \quad 1 \quad 0 \quad 0]^T \tag{36}$$

For $\delta u_y = -0.001$, the corresponding variations in the joint torques and joint angles are

$$\begin{aligned} \delta\tau &= 10^{-3}\,[-2 \quad -2 \quad 0 \quad 0 \quad 2 \quad 2 \quad 0 \quad 0]^T \\ \delta q &= 10^{-3}\,[0 \quad 1 \quad -1 \quad 0 \quad 0 \quad -1 \quad 1 \quad 0]^T, \end{aligned} \tag{37}$$

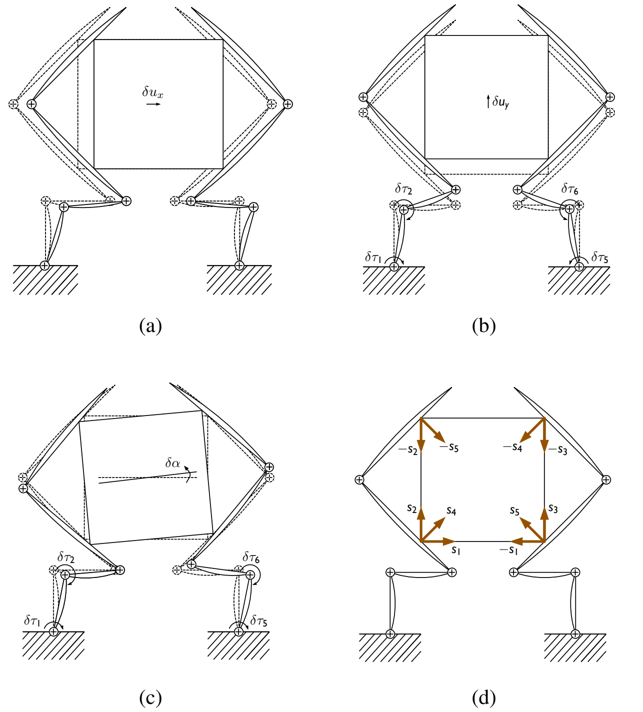

Fig. 5: Graphic representations of numerical results for the internal power grasp variations.

and can be interpreted as the second column of $\Gamma_{\tau k}$ and Γ_{qk}: pictorial representations are sketched in Fig. 5. For an object rotation $\delta u_\alpha = 0.001$, Fig. 5c, the variations in the joint torques and joint angles are then

$$\begin{aligned} \delta\tau &= 10^{-3}\,[3 \quad 3 \quad 0 \quad 0 \quad 3 \quad 3 \quad 0 \quad 0]^T\,, \\ \delta q &= 10^{-3}\,[-1.5 \quad 1 \quad 1.5 \quad 0 \quad -1.5 \quad 1 \quad 1.5 \quad 0]^T\,, \end{aligned} \tag{38}$$

corresponding to the third column of $\Gamma_{\tau k}$ and Γ_{qk}, respectively. The five possibilities for the squeeze are sketched in Fig. 5d, where the i^{th} couple of forces s_i and $-s_i$ correspond to the i^{th} achievable squeeze. The numerical results for $\delta\tau$ and δq are omitted here for space limitations.

2) A Synergy in the Power Grasp: underactuating the hand with the following single-column synergy matrix

$$S = \begin{matrix} [-0.6500 & 0 & -0.3200 & -0.4000 \\ 0.6500 & 0 & 0.3200 & 0.4000]^T \end{matrix} \tag{39}$$

we get: $n_{s_q} = 1$, $n_{c_o} = 3$ and $n_i = n_k = n_{s_t} = 0$. To unveil a property of this synergy we can study how the system responds to different inputs. In the absence of an external interaction, δw_e^b=0, and with an unitary synergistic actuation, $\delta\sigma_r$=1, contact forces and object displacements become

$$\delta f_{c^h}^{c^o} = \begin{matrix} [0.5043 & 0.5043 & 0.5043 & -0.5043 \\ -0.5043 & 0.5043 & -0.5043 & -0.5043]^T, \end{matrix} \tag{40}$$

$$\delta u = [0 \quad 0 \quad 0]^T\,, \tag{41}$$

indicating that we are (purely) squeezing the object along both diagonals. It is worth noting that the above synergy was constructed by summing two columns of $\Gamma_{q_r s_q}$ in (26) for the fully-actuated system.

VI. CONCLUSIONS

In this work, we have presented a framework to model and study the structural properties of a grasp by a general robotic hand in a quasi-static setting. The hand can be arbitrarily underactuated (e.g., via synergistic actions), and can be endowed with compliant characteristics both in the fingerpads and in the actuation system. The mathematical model defined here considers also the derivatives of the Jacobian matrix and employs a right-invariant definition of the grasp matrix, since other definitions may lead to potentially misleading conclusions. An algorithm based on the extensive employment of the RREF can be applied to translate the quest for interesting behaviors in the hand object system to some predefined sparsity patterns in a suitably defined global system matrix. In order to assess the validity of the proposed method, some numerical tests have been presented for two different grasps: a precision grasp and a power grasp. These demonstrated: (i) the importance of the terms originated by the derivatives of the Jacobian matrix in the presence of a preload contact force to obtain physically meaningful results, and (ii) the influence of the synergistic actuation on the structural properties of a grasp configuration. An exhaustive classification of general manipulation behaviours (all feasible sparsity patterns) is subject of the ongoing research.

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