Force Distribution in Multiple Whole-Limb Manipulation

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Abstract

This paper deals with robotic systems composed of multiple actuated limbs (such as arms, fingers, or legs) cooperating in the manipulation of an object. The problem of decomposing the system of contact forces exerted between the robot limbs and the object, in order to apply a desired resultant force on the object (and/or to resist to external disturbances) is studied. Enveloping (or "whole-limb") manipulation operations that exploit any part of the limbs to contact the object are considered. The peculiarity of such systems is that contacts occurring on links with limited mobility (such as the inner links of a robot arm or hand), and even on fixed links (a robot chest or palm), are possible. Among the many consequences of this characteristic, the one studied in this paper is that only some of the internal grasping forces may be actively controlled to increase grasp stability.

1 Introduction

Multiple robot manipulation systems are becoming increasingly popular in various applications. Typical examples are robot arms that cooperate in carrying a heavy or bulky object, robot hands that use several fingers for dextrous manipulation tasks, and deambulating vehicles that use their legs to negotiate difficult terrains. Equipping multiple manipulation systems with the ability to use any part of their links for contacting and manipulating objects is one way of further enhancing their capabilities and applicability potentials. As often occurs in robotics, this idea comes directly from the observation of human and animal examples. The arms and chest of a man carrying large objects, his hand used to firmly grasp an object between the phalanges of fingers and the palm ("power grasping"), or the limbs of an ape when climbing a tree, provide us with the evidence of the usefulness of such enveloping manipulation in nature. Trinkle [1987] studied planning techniques for enveloping, frictionless grasping. Robotic devices intended to exploit the whole-arm manipulation idea have been pioneered by the MIT WAM (Whole Arm Manipulator) project [Salisbury, 1987]. A dextrous hand using all its parts (including the inner phalanges and the palm) to achieve robust power grasps and high manipulability has been proposed by Vassura and Bicchi [1989]; Mirza and Orin [1990] described a multiple arm manipulation system (DIGITS), and discussed the improved robustness of power grasping.

A characteristic of enveloping manipulating systems is their use of links that have only limited mobility: for example, a hand's palm has no mobility at all. Thus, such systems are intrinsically defective, i.e., they possess fewer degrees-of-freedom than necessary to achieve arbitrary configurations in their operational space. Defectivity of enveloping manipulation systems poses new problems in the kineto-statics and dynamics analysis, that cannot be dealt with by standard methods ([Bicchi and Prattichizzo, 1992]). This paper is devoted to the investigation of a particular aspect of the analysis of enveloping manipulation systems, i.e. the problem of understanding which contact forces can be actively used to improve the grasp on the object.

2 Background

The problem of controlling contact forces in a multiple manipulation system such as a hand, a pair of cooperating robot arms, or a legged vehicle, has been traditionally considered in the assumption that every single finger (arm, or leg: in the following, we will refer to "hands" generically) has full mobility in its task space. This assumption greatly simplifies the problem, by allowing to separately deal with the analysis of the distribution of grasp force among the contacts, and with the control of the joint torques that realize desirable contact forces. In this section we briefly review the background on grasp analysis techniques, in order to highlight what new problems are posed by enveloping systems.

Let for instance an object be grasped by means of n contacts and let the components of contact forces and moments on the object form a vector $\mathbf{t} \in \mathbb{R}^t$. Consider the task of resisting an external force $\mathbf{f} \in \mathbb{R}^3$ and moment $\mathbf{m} \in \mathbb{R}^3$ applied upon the object (the task of steering an object along a desired trajectory is equivalent once the inertial load corresponding to the specified acceleration and velocity profile is determined). The force and moment balance equation for the object can be written

$$\mathbf{w} = -\mathbf{G}\mathbf{t}, \tag{1}$$

where $\mathbf{w} = (\mathbf{f}^T\mathbf{m}^T)^T \in \mathbb{R}^6$ is the load wrench, and $\mathbf{G} \in \mathbb{R}^{6\times t}$ is the grasp matrix. The general solution

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consists (assuming that $\mathbf{w} \in \mathcal{R}(\mathbf{G})$) of the sum of a particular and a homogeneous solution,

$$\mathbf{t} = -\mathbf{G}^R\ \mathbf{w} + \mathbf{A}\mathbf{x},$$

where $\mathbf{G}^R$ is a generic right-inverse of the grasp matrix, and $\mathbf{A} \in \mathbb{R}^{t\times h}$ is a matrix whose columns form a basis of the nullspace of $\mathbf{G}$ (denoted with $\mathcal{N}(\mathbf{G})$). The coefficient vector $\mathbf{x} \in \mathbb{R}^h$ parameterizes the homogeneous part of the solution eq.(2): for any choice of $\mathbf{x}$, a vector of contact forces results that equilibrates the desired load. Grasp optimization techniques (see e.g. [Nakamura, Nagai and Yoshikawa, 1989]), can be formulated by defining a cost and constraint functions so as to, e.g., avoid contact slippage and minimize consumption of power in the joint actuators. Several algorithms have been proposed to find the optimal $\hat{\mathbf{x}}$. The corresponding $\hat{\mathbf{t}} = -\mathbf{G}^R\mathbf{w} + \mathbf{A}\hat{\mathbf{x}}$ is the optimal force distribution among contacts with respect to the criterion adopted in the design of V. In non-defective systems, $\hat{\mathbf{t}}$ is applied by the fingers under some type of force control technique. However, according to the relationship between the contact forces on the fingers and the vector $\tau \in \mathbb{R}^q$ of joint actuator torques, $\tau = \mathbf{J}^T\mathbf{t}$. ($\mathbf{J} \in \mathbb{R}^{t\times q}$ is the "Jacobian" of the robot), when the robot system is defective ($q \geq$ rank $(\mathbf{J}) < t$) there is no guarantee that the optimal contact forces can actually be realized by the robot. In other words, complete (output) controllability of internal forces arises is not achieved in those cases. While the controllability concept can be investigated in a dynamical model of grasping, in this paper we undertake a quasi-static analysis meant to answer the question, "what internal forces at equilibrium are modifiable at will, when inputs are joint torques?".

Consider for example the grasp of the object depicted in fig.1-a by means of three contacts placed in $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathbf{c}_3$. Intuitively, there are three possible independent combinations of contact forces giving homogeneous solutions to the grasp equations, namely those pushing or pulling the object along the edges of the so-called "grasp triangle" (fig.1-b). Any pair of these "internal" forces or their combinations may be used, for instance, to squeeze the object and decrease the danger of slippage. However, if the grasp is to be realized by the simple single-joint gripper shown in fig.1-c, it appears that some configuration of internal forces may not be feasible (for instance, opposing forces in the direction $\mathbf{c}_2 - \mathbf{c}_3$ as shown in the uppermost part of fig.1-b). In order to solve the force decomposition problem for general manipulation systems, a more accurate analysis is therefore necessary, which takes into account the kinematics and the deformability of the manipulation system. To incorporate contact constraints in the model, relative displacements between the object and the links at the contact centroids must be considered. Therefore, we introduce n reference frames ${}^oC_i$ fixed w.r.t. the object and centered at the contact point $\mathbf{c}_i$; and n reference frames ${}^mC_i$, each fixed w.r.t. the link that touches the object in $\mathbf{c}_i$, and centered in $\mathbf{c}_i$. Corresponding to a small displacement $\Delta\mathbf{r}$ and rotation $\Delta\phi$ of the object

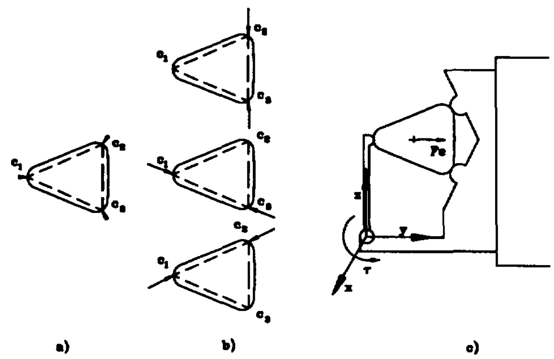

Figure 1: A simple example of whole-hand grasp between the palm and a single-joint finger.

w.r.t. the base frame (summarized in the "twist" vector $\Delta\mathbf{u} = (\Delta\mathbf{r}^T, \Delta\phi^T)^T$), frames ${}^oC_i$ undergo a displacement $\Delta^o\mathbf{c}_i$ and rotation $\Delta^o\phi_i$ whose relationship with $\Delta\mathbf{u}$ can be derived by the virtual work principle as

$$\Delta^o\mathbf{x} = \mathbf{G}^T\ \Delta\mathbf{u}, \qquad (2)$$

$$\Delta^o\mathbf{x} = (\Delta^o\mathbf{c}_1^T, \ldots, \Delta^o\mathbf{c}_n^T, \Delta^o\phi_1^T, \ldots, \Delta^o\phi_n^T)^T.$$

Analogous is the relationship between small displacements of the joints $\Delta\mathbf{q}$ and the displacements $\Delta^m\mathbf{c}_i$ and rotations ${}^m\Delta\phi_i$ of the contact frames ${}^mC_i$:

$$\Delta^m\mathbf{x} = \mathbf{J}\ \Delta\mathbf{q}, \qquad (3)$$

$$\Delta^m\mathbf{x} = (\Delta^m\mathbf{c}_1^T, \ldots, \Delta^m\mathbf{c}_n^T, \Delta^m\phi_1^T, \ldots, \Delta^m\phi_n^T)^T.$$

Contact constraints impose that certain components of the relative displacements $\Delta^o\mathbf{x} - \Delta^m\mathbf{x}$ are selectively opposed by reaction forces, depending upon the type of contact. Several types of contact models can be used to describe the interaction between the links and the object, among which the most useful are probably the point-contact-with-friction (also called "hard-finger") model, the "soft-finger" model, and the complete-constraint (or "very-soft-finger") model. For a description of these models, see e.g. [Mason and Salisbury, 1985]. Contact constraints can be expressed in terms of a suitable selection matrix $\mathbf{H}$ as

$$\mathbf{H}(\Delta^m\mathbf{x} - \Delta^o\mathbf{x}) = 0, \qquad (4)$$

All relationship considered so far are valid for a rigid-body model of the robot system. However, the force distribution problem for general systems is an under-determined problem of statics: to solve the indeterminacy, the rigid body model is inadequate. We have therefore to consider a more accurate model, taking into account the elastic elements that are involved in the system. This can be conceptually done by introducing a set of "virtual springs" [Hanafusa and Asada, 1977] interposed between the links and the object at the contact points, such that the elastic relationship between the relevant components of the relative displacements $\Delta^o\mathbf{x} - \Delta^m\mathbf{x}$ and the corresponding components of contact forces can be written as

$$\mathbf{t} = \mathbf{KH}(\Delta^m\mathbf{x} - \Delta^o\mathbf{x}) + \mathbf{t}_o, \qquad (5)$$

where $\mathbf{t}_o$ is the contact force in the reference configuration $\Delta^m\mathbf{x} = \Delta^o\mathbf{x} = 0$. The stiffness matrix $\mathbf{K} \in \mathbb{R}^{t\times t}$ incorporates the structural elasticity of the object and of the fingers, and the stiffness of joint servos [Mason and Salisbury, 1985]. As a consequence of its physical nature, $\mathbf{K}$ can be assumed non-singular. A detailed and comprehensive study on the evaluation and the realization of desirable stiffness matrices with articulated hands has been presented by Cutkosky and Kao [1989]. It should be noted that, since $\mathbf{K}$ incorporates the stiffness of the joint position controllers, the displacement vector $\Delta\mathbf{q}$ must be interpreted as the change in the input reference for position controllers.

3 The homogeneous solution

Internal forces, i.e. self-balanced contact forces that have no effect on the global motion of the manipulated object but significantly affect the grasp stability, correspond to homogeneous solutions of eq.(1). However, as suggested above, not all homogeneous solutions may be actively controlled by using joint variables as inputs. Internal contact forces that are not actively realizable through joint control may still be present in a system, due to its initial conditions - e.g., they may have been set by prestressing elastic elements in the manipulation system. In this section we propose a decomposition of the homogeneous subspace in a subspace of active, internal contact forces and a subspace of passive (preload), internal contact forces.

3.1 Active Internal Forces

Consider an equilibrium configuration of the manipulation system under an external load $\mathbf{w}_o$, and denote with $\mathbf{q}_o$ and with $\mathbf{t}_o$ the joint positions and the contact forces in such reference configuration, respectively. By modifying the joint reference position by $\Delta\mathbf{q}$, the equilibrium configuration of the object, still subject to $\mathbf{w}_o$, is changed by $\Delta\mathbf{u}$. Correspondingly, contact forces are $\mathbf{t} = \mathbf{t}_o + \Delta\mathbf{t}$. From $\mathbf{Gt} = \mathbf{w}_o$ follows that $\Delta\mathbf{t} \in \mathcal{N}(\mathbf{G})$. We define *active* those internal contact forces $\Delta\mathbf{t}$ that correspond to controllable modifications of the system configuration, and let $\mathcal{F}_{hr} \in \mathbb{R}^t$ denote the set of all active $\Delta\mathbf{t}$'s.

Proposition 1 *The set of active internal forces $\mathcal{F}_{hr}$ is a subspace of $\mathbb{R}^t$, i.e., every active internal force can be written as the product of a basis matrix $\mathbf{E}$ times an arbitrary coefficient vector $\mathbf{y}$ of suitable dimension.*

Proof. Consider a system in the equilibrium configuration described by $\mathbf{w}_o$, $\mathbf{q}_o$, $\mathbf{t}_o$, and let $\delta\mathbf{u}$ be a displacement of the object which is compatible with all the constraints imposed by contacts with the robot links (i.e., $\delta\mathbf{u}$ is a virtual displacement of the object). Applying the principle of virtual work (P.V.W.) and eq.(1), we have immediately

$$\mathbf{w}_o^T\delta\mathbf{u} = \mathbf{t}_o^T\mathbf{G}^T\delta\mathbf{u} = 0, \quad \forall\delta\mathbf{u}.$$

By imposing joint displacements $\Delta\mathbf{q}$, the equilibrium configuration is perturbed. A new equilibrium under the same external force $\mathbf{w}_o$ will be reached on condition that the P.V.W. is satisfied:

$$\mathbf{w}_o^T\delta\mathbf{u} = (\mathbf{t}_o + \Delta\mathbf{t})^T\mathbf{G}^T\delta\mathbf{u} = \Delta\mathbf{t}^T\mathbf{G}^T\delta\mathbf{u} = 0, \quad \forall\delta\mathbf{u}.$$

From eq.(5), $\Delta\mathbf{t} = \mathbf{K}(\mathbf{J}\Delta\mathbf{q} - \mathbf{G}^T\Delta\mathbf{u})$. After substitution, the P.V.W. condition is

$$\Delta\mathbf{q}^T\mathbf{J}^T\mathbf{K}^T\mathbf{G}^T\delta\mathbf{u} = \Delta\mathbf{u}^T\mathbf{G}\mathbf{K}^T\mathbf{G}\delta\mathbf{u} \quad \forall\delta\mathbf{u}$$

which implies

$$\mathbf{GKJ}\Delta\mathbf{q} = \mathbf{GKG}^T\Delta\mathbf{u}, \qquad (6)$$

and hence

$$\begin{aligned}\Delta\mathbf{u} &= (\mathbf{GKG}^T)^{-1}\mathbf{GKJ}\Delta\mathbf{q} \qquad (7)\\ \Delta\mathbf{t} &= \mathbf{K}(\mathbf{J}\Delta\mathbf{q} - \mathbf{G}^T(\mathbf{GKG}^T)^{-1}\mathbf{GKJ}\Delta\mathbf{q}) =\\ &= (\mathbf{I} - \mathbf{G}_K^R\mathbf{G})\mathbf{KJ}\Delta\mathbf{q} \qquad (8)\end{aligned}$$

where $\mathbf{G}_K^R$ stands for the $\mathbf{K}$-weighted pseudo-inverse of $\mathbf{G}$. Therefore, all active internal forces can be expressed as

$$\tilde{\mathbf{t}} = \mathbf{Ey}, \qquad (9)$$

where the columns of the $t \times e$ matrix $\mathbf{E}$ form a basis of the range of $(\mathbf{I} - \mathbf{G}_K^R\mathbf{G})\mathbf{KJ}$. □

The vector $\mathbf{y} \in \mathbb{R}^e$ is comprised of e free variables among which the $\hat{\mathbf{y}}$ corresponding to an optimal grasp force distribution can be chosen by means of suitable cost functions and optimization routines. Note that in general $e \leq h$, i.e. active internal forces are "fewer" than internal forces, corresponding to intuition. Also, $e \leq q$, i.e. no more independent active internal forces can be controlled than are joints in the system.

From a computational point of view, the algorithm sketched in the proof of Proposition 3 to evaluate the desired basis matrix $\mathbf{E}$ is not optimal, since it entails the explicit calculation of the right-inverse $\mathbf{G}_K^R$. A more efficient algorithm, which also provides further insight in the problem, can be derived by rewriting eq.(6) as

$$\mathbf{G}\ (\mathbf{KJ}\ \Delta\mathbf{q}\ - \mathbf{KG}^T\Delta\mathbf{u}) = 0,$$

or, equivalently, as

$$\mathbf{Ax} = \mathbf{KJ}\Delta\mathbf{q} - \mathbf{KG}^T\Delta\mathbf{u}.$$

This equation can be recast in block matrix form as

$$\begin{bmatrix}\mathbf{A} & -\mathbf{KJ} & \mathbf{KG}^T\end{bmatrix}\begin{pmatrix}\mathbf{x}\\ \Delta\mathbf{q}\\ \Delta\mathbf{u}\end{pmatrix} = 0. \qquad (10)$$

Put $\mathbf{Q} = \begin{bmatrix}\mathbf{A} & -\mathbf{KJ} & \mathbf{KG}^T\end{bmatrix} \in \mathbb{R}^{t\times(h+q+6)}$, and let $\mathbf{B} \in \mathbb{R}^{(h+q+6)\times b}$ be a matrix whose columns span the nullspace of $\mathbf{Q}$ (whose nullity is b). Finally, partition $\mathbf{B}$ as $\mathbf{B} = [\mathbf{B}_1^T\ \mathbf{B}_2^T\ \mathbf{B}_3^T]^T$, where $\mathbf{B}_1 \in \mathbb{R}^{h \times b}$, $\mathbf{B}_2 \in \mathbb{R}^{q \times b}$, and $\mathbf{B}_3 \in \mathbb{R}^{6 \times b}$. The subspace under investigation is thus obtained as

$$\mathcal{F}_{hr} = \mathcal{R}(\mathbf{A}\mathbf{B}_1).$$

and the matrix $\mathbf{E}$ is obtained by using only the independent columns of $\mathbf{A}\mathbf{B}_1$. This method, though seemingly complex, is numerically more efficient and robust than the previously presented, since it avoids any matrix inversion. Further, by using eq.(10) it can be easily calculated which joint reference displacements must be commanded if a desired active internal force $\Delta\hat{\mathbf{t}} = \mathbf{E}\hat{\mathbf{y}}$ is to be applied:

$$\Delta\hat{\mathbf{q}} = \mathbf{B}_2(\mathbf{A}\mathbf{B}_1)^+\mathbf{E}\hat{\mathbf{y}} \tag{11}$$

The equilibrium position of the object is correspondingly displaced by $\Delta\hat{\mathbf{u}} = \mathbf{B}_3(\mathbf{A}\mathbf{B}_1)^+\mathbf{E}\hat{\mathbf{y}}$.

3.2 Preload internal forces

As mentioned above, in general manipulation systems there may be internal contact forces that can not be actively controlled by means of joint displacements. Therefore, these forces will remain constantly equal to their initial value. In a mechanical jig, such forces can be set once and for all as a preload condition, for instance for preventing slippage. Although in robotic systems it may be unlikely to encounter such preload forces, their analysis is an interesting completion to the study above.

Let $\mathcal{F}_{ho} \in \mathbb{R}^t$ denote the subspace of internal, passive (preload) contact forces, and let the subspace of contact forces that the manipulation system can exert on the object with zero joint torques be

$$\mathcal{F}_o = \{\mathbf{t} \in \mathbb{R}^t \,|\, \mathbf{J}^T\mathbf{t} = 0\} \equiv \{\mathbf{t} \in \mathbb{R}^t \,|\, \mathbf{t} = \mathbf{C}\mathbf{z}_2, \forall \mathbf{z}_2 \in \mathbb{R}^k\}$$

where $\mathbf{C} \in \mathbb{R}^{t \times k}$ is a matrix whose column form a basis of the nullspace of $\mathbf{J}^T$ (whose nullity is k). The preload force subspace is thus given by

$$\mathcal{F}_{ho} = \mathcal{F}_h \cap \mathcal{F}_r = \mathcal{R}(\mathbf{A}) \cap \mathcal{R}(\mathbf{C}).$$

Proposition 2 *The set of passive internal forces $\mathcal{F}_{ho}$ is a subspace of $\mathbb{R}^t$, i.e., every passive internal force can be written as the product of a basis matrix $\mathbf{P}$ times an arbitrary coefficient vector $\mathbf{z}$ of suitable dimension.*

Proof. Since the desired set is the intersection of the range space of matrices $\mathbf{A}$ and $\mathbf{C}$, it is a subspace. To evaluate a basis, consider the equation $\mathbf{A}\mathbf{z}_1 = \mathbf{C}\mathbf{z}_2$, or, in matrix form,

$$[\mathbf{A}\ \ -\mathbf{C}]\begin{pmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{pmatrix} = 0. \tag{12}$$

Let $\mathbf{Q}_o = [\mathbf{A}\ -\mathbf{C}] \in \mathbb{R}^{t \times (h+k)}$, and let $\mathbf{B}_o \in \mathbb{R}^{(h+k) \times d}$ be a matrix whose columns span the nullspace of $\mathbf{Q}_o$ (whose nullity is d). Finally, partition $\mathbf{B}_o$ as $\mathbf{B}_o = [\mathbf{B}_{o1}^T\ \mathbf{B}_{o2}^T]^T$, where $\mathbf{B}_{o1} \in \mathbb{R}^{h \times d}$, and $\mathbf{B}_{o2} \in \mathbb{R}^{k \times b}$. The desired subspace is thus obtained as

$$\mathcal{F}_{ho} = \mathcal{R}(\mathbf{A}\mathbf{B}_{o1}).$$

Therefore, all possible preload forces can be expressed as

$$\mathbf{t} = \mathbf{P}\mathbf{z}, \tag{13}$$

where the columns of the $t \times p$ matrix $\mathbf{P}$ form a basis of the range of $\mathbf{A}\mathbf{B}_{o1}$, and $\mathbf{z} \in \mathbb{R}^p$ parameterizes the preload subspace. □

From the definition of the particular, active and preload homogeneous force subspaces follows

$$\mathcal{R}(\mathbf{P}) \oplus \mathcal{R}(\mathbf{E}) = \mathcal{N}(\mathbf{G}) \tag{14}$$

$$\mathcal{R}(\mathbf{P}) \oplus \mathcal{R}(\mathbf{E}) \oplus \mathcal{R}(\mathbf{G}_K^R) = \mathbb{R}^t. \tag{15}$$

4 Examples

In this section we will illustrate the above discussed algorithms and show how the manipulator kinematics and elasticity properties play an important role in the analysis of the grasp when general manipulation systems are considered. In order to do that, we will consider the grasp of the object depicted in fig.1-a by means of four different manipulation systems.

Let the coordinates of contact points be $\mathbf{c}_1 = (0\ 0\ 2a)^T$; $\mathbf{c}_2 = (0\ 2a\ 3a)^T$; $\mathbf{c}_3 = (0\ 2a\ a)^T$, and the corresponding unit normal vectors be $\mathbf{n}_1 = (0\ 1\ 0)^T$; $\mathbf{n}_2 = (0\ -\frac{\sqrt{3}}{2}\ -\frac{1}{2})^T$; and $\mathbf{n}_3 = (0\ -\frac{\sqrt{3}}{2}\ \frac{1}{2})^T$. All contacts are modeled as "soft-finger". Accordingly, the dimension of composite contact force/torque vectors $\mathbf{t}$ is $t = 12$, and a basis of the null-space of $\mathbf{G}$ is provided by the columns of the matrix $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & -2 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & -4a & \frac{2}{\sqrt{3}}a & 1 \\ 0 & 0 & 0 & 4a & \frac{2}{\sqrt{3}}a & 1 \end{bmatrix}$$

Note that the first three columns correspond to contact forces taken two at a time and opposing each other along the edges of the grasp triangle. The presence of friction torques at the soft-finger contacts produces the last three columns of $\mathbf{A}$. The stiffness matrix $\mathbf{K}$ will be evaluated in each case according to Cutkosky and Kao [1989], as

$$\mathbf{K} = (\mathbf{C}_s + \mathbf{J}\mathbf{C}_q\mathbf{J}^T)^{-1},$$

where $\mathbf{C}_s$ is the structural compliance matrix, and $\mathbf{C}_q$ is the servo compliance matrix. The structural compliance is due e.g. to the flexibility of links and mechanical transmission, or to soft gripping surfaces. In

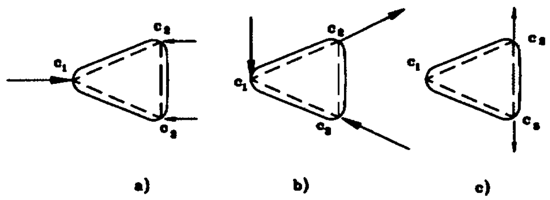

Figure 2: Active and preload internal forces for the grasp of fig.1

our example, $\mathbf{C}_s$ is assumed diagonal, with $\mathbf{C}_{s_{j,j}} = 0.05$ mm/N for linear virtual springs $(1 \leq j \leq 9)$, and $\mathbf{C}_{s_{j,j}} = 0.01$ deg./Nmm for rotational virtual springs $(10 \leq j \leq 12)$. On the other hand, assuming that the q joints are controlled with q independent PD servos with proportional gain $k_p = 100.0$ Nmm/deg., we have $\mathbf{C}_q = \frac{1}{k_p}\mathbf{I}_q$.

4.1 Simple gripper

Consider the simple one-joint gripper of fig.1-c. The joint axis is $\mathbf{s}_1 = (1\ 0\ 0)^T$, and its origin $\mathbf{o}_1 = (0\ 0\ 0)^T$. The jacobian matrix in this case is $\mathbf{J}^T = (0\ -2a\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$. As intuitively clear, the subspace of active internal forces is one-dimensional in this example, and the preload force subspace is 5-dimensional:

$$\mathbf{E} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \mathbf{P} = \begin{bmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & -\sqrt{2} & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}a & 0 & \sqrt{3} \\ -4a & 0 & a & 0 & 1 \\ 4a & 0 & a & 0 & 1 \end{bmatrix}$$

Recall that each column represents a combination of contact forces $\mathbf{t}_i$ and normal torques $m_i\mathbf{n}_i$ at the contact points, arranged as $(\mathbf{t}_1^T\ \mathbf{t}_2^T\ \mathbf{t}_3^T\ m_1\ m_2\ m_3)^T$. The only set of internal forces that can be modified at will is represented in fig.2.a. Fig.2-b and fig.2-c represent two of the basic combinations of passive internal forces (columns 2 and 4 of $\mathbf{P}$, respectively), that cannot be modified by joint control.

4.2 Two-joint hand

Consider the two-joint hand of fig.3.a, which employs the two links and the palm to grasp the object of fig.1-a. Joint axes are $\mathbf{s}_1 = \mathbf{s}_2 = (1\ 0\ 0)^T$, and the origins are $\mathbf{o}_1 = (0\ 0\ 0)^T$, and $\mathbf{o}_2 = (0\ 0\ 4a)^T$. The subspace of active, internal forces and preload contact forces are now 2- and 4-dimensional, respectively, and

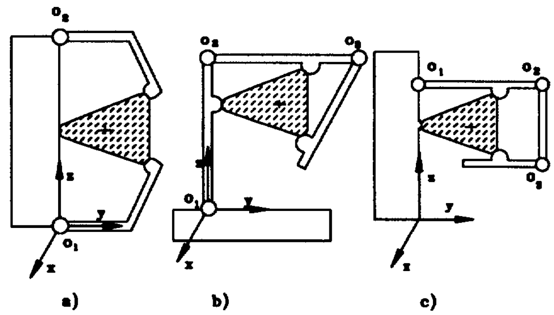

Figure 3: Three different manipulators grasping the same object.

their basis matrices are

$$\mathbf{E} = \begin{bmatrix} 0 & 0 \\ -2 & 0 \\ 0 & -2 \\ 0 & 0 \\ 1 & 2 \\ 6 & 1 \\ 0 & 0 \\ 1 & -2 \\ -6 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} ; \mathbf{P} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -\sqrt{2} & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & \sqrt{3} \\ -4 & 0 & 1 & 1 \\ 4 & 0 & 1 & 1 \end{bmatrix}$$

4.3 Three-joint finger.

Consider now the three-joint finger depicted in fig.3.b, where $\mathbf{s}_1 = \mathbf{s}_2 = \mathbf{s}_3 = (1\ 0\ 0)^T$, and $\mathbf{o}_1 = (0\ 0\ 0)^T$, $\mathbf{o}_2 = (0\ 0\ 3a)^T$, and $\mathbf{o}_3 = (0\ 3a\ 3a)^T$. The dimension of the active and preload force subspaces are not changed in this case:

$$\mathbf{E} = \begin{bmatrix} 0 & 0 \\ -2 & 0 \\ 0 & 5 \\ 0 & 0 \\ 1 & -5 \\ 0 & 1 \\ 0 & 0 \\ 1 & 5 \\ 0 & -6 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} ; \mathbf{P} = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 1 & 0 & -\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}a & \sqrt{3} \\ -4a & 0 & a & 1 \\ 4a & 0 & a & 1 \end{bmatrix}$$

4.4 Three-joint finger and palm.

If the object of fig.1-a is grasped by a three-joint finger and the palm of a hand such as depicted in fig.3.c, a three-dimensional subspace of active internal forces can be obtained. In fact, assuming in this example $\mathbf{s}_1 = \mathbf{s}_2 = \mathbf{s}_3 = (1\ 0\ 0)^T$, $\mathbf{o}_1 = (0\ 0\ 3a)^T$,

$\mathbf{o}_2 = (0\ 3a\ 3a)^{\mathrm{T}}$, and $\mathbf{o}_3 = (0\ 3a\ a)^{\mathrm{T}}$, the **E** and **P** basis matrices result

$$\mathbf{E} = \begin{bmatrix} 0 & 0 & 0 \\ -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \mathbf{P} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -\sqrt{2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}}a & \sqrt{3} \\ -4a & a & 1 \\ 4a & a & 1 \end{bmatrix}$$

4.5 Kerr and Roth's example

In an important early paper on grasp optimization, Kerr and Roth [1986] discuss an example grasp by two fingers (fig. 4). They use linear programming techniques to choose the optimal combination of forces in the nullspace of the grasp matrix, in our notation:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \sqrt{2}/2 & 0 \\ 0 & 0 \\ 0 & 0 \\ \sqrt{2}/2 & 0 \\ 0 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 \end{bmatrix}.$$

However, the manipulation system is apparently defective, and only the span of the first column of **A** results actively modifiable through joint torque commands. Therefore, the optimization search should have been performed inside that span (corresponding to that of matrix **E** by the method above), to avoid possibly unfeasible results; moreover, reduction of the dimensionality of the search space is of great advantage for computational issues.

5 Conclusion

In this paper the problem of force decomposition in general manipulation systems, including multiple whole-limb cooperating manipulators, has been considered. An attempt is made at explaining the geometric structure of the vector space of contact forces and torques that are mutually exerted between the manipulation system links and the manipulated object. Algorithms for the determination of a vector basis for such subspaces are also provided. These results, particularly those concerning the description of active internal forces, are necessary for planning and control of optimal grasp algorithms with whole-limb manipulation systems.

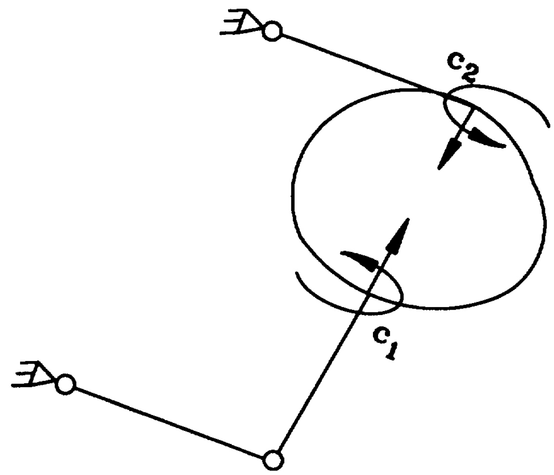

Figure 4: Kerr and Roth's [1986] example no.1.

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