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## 6 CONCLUSIONS

In this paper we presented comparison between the Moore-Penrose inverse method and the generalized inverse method for the decomposition of forces and moments into the external and internal forces and moments. We derived equations to relates the load sharing coefficients to the external and internal forces and moments. Through this we clarified independent parameters to be adjusted for the two cooperative manipulators to hold a common object robustly. The robust holding problem was thus formulated as a quadratic programming problem. Numerical examples showed that the robot accommodates forces and moments intelligently in order not to drop the object when the external forces and moments are applied. In the same manipulation done by human beings "on-line" decision making is needed not to drop the object and the one being done after this looks like "intelligent." This paper showed that the same task can be implemented by machines using the quadratic programming. We believe, therefore, that the dexterity of the manipulator is by no means mysterious. It is just one of our technological problems. The problem of determining those parameters should constitute a part of intelligent control for dextrous manipulation.

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# New Issues in the Kineto-Statics, Dynamics, and Control of Whole-Hand Manipulation

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#### Abstract

In the observation of human and animal grasping and manipulation, one frequently notes the fundamental role played by the inner parts of the hand (palm and proximal phalanges) to enhance both the stability of the grip and the versatility of operation. To transfer this enhanced dexterity into robotic devices, researchers have designed hands with the ability of using inner surfaces for contact with the object, and capable of sensing contact interactions. On the other hand, most of the analytical studies of dextrous manipulation have been carried out so far in the simplifying assumption that objects are manipulated by fingertips only.

In this paper, we will discuss what new problems are introduced by the use of the whole hand in the kineto-static and dynamic analysis of dextrous manipulation. The introduction of sensor feedback is an important means for overcoming difficult problems arising from the complexity of the model. A description of research efforts in this field will be given, and some recent results will be presented.

## 1 Introduction

Robots that are currently employed in industrial applications and in most research laboratories can be regarded as very ineffective mechanical devices, at least as far as the ratio between their weight and the payload is concerned. The reason why such massive and costly machines can only manipulate very small and relatively lightweight objects is that only their end-effectors are used for manipulation. Being the end-effector at the distal end of the arm, the effects of forces applied at the end-effector are extremely significant on the inner links and joints. The effective exploitation of the inner links of the arm would allow a major enhancement of the capabilities of robot manipulators, by allowing them to negotiate large and bulky objects with dexterity. In the case of autonomous locomotion, the use of inner parts could afford present legged systems with the capability of roving and climbing over difficult obstacles and terrains. However, it is probably in the field of dextrous manipulation and grasping that this concept may have its most important consequences.

In the observation of human and animal grasping and manipulation, the fundamental role played by the inner parts of the hand (palm and proximal phalanges) to enhance both the stability of the grip and the versatility of operation can be frequently observed. To transfer this enhanced dexterity into robotic devices, researchers have conceived hands with the ability of using inner surfaces for contact with the object, and capable of sensing contact interactions (see e.g. [Melchiorri and Vassura, 1992]). Mirza and Orin [1990] showed the largely increased holding capability of a robot hand exploiting its inner links and palm for grasping. On the other hand, most of the analytical studies of dextrous manipulation have been carried out so far in the simplifying assumption that objects are manipulated by fingertips only, and as such they do not apply directly to whole-hand manipulation. In this paper, we discuss what new problems in the formulation of the kineto-static and dynamic problem for robot hands are posed by the use of whole-hand manipulation.

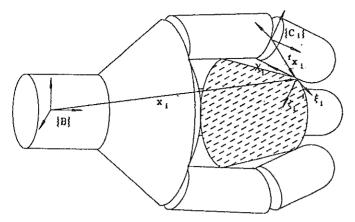


Figure 1: Whole-hand manipulation of an object allows contacts on the inner phalanges and palm

#### Kinematics

The model of the hand we assume is comprised of an arbitrary number of fingers (i.e. simple chains of links -phalanges -, connected through rotoidal or prismatic joints), and of an object, which is in contact with some or all of the phalanges (see fig. 1). We define the vector q as a vector of generalized coordinates, completely describing the configuration of the fingers; and the vector u as a generalized coordinate vector for the object. Contacts represent a particular kind of kinematic constraint on the allowable configurations of the system, and make for the most of the differences in the analysis of dextrous manipulation from conventional robotics. Contact constraints are typically unilateral, non-holonomic constraints on the generalized coordinates system, written in general in the form

$$C(q, \dot{q}, u, \dot{u}) \ge 0.$$
 (1)

The inequality relationship reflects the fact that contact can be lost if the contacting bodies are brought away from each other. This involves an abrupt change of the structure of the model under consideration. To avoid analytical difficulties, it is usually assumed that the manipulation is studied during time intervals when constraints hold with the equal sign. The constraint relationship eq.(1) is not in general integrable, i.e., it cannot be expressed in terms of q and u only: integrable constraints are called "holonomic". Holonomic constraints between generalized coordinates reduce the number of independent coordinates necessary to describe the system configuration (degrees of freedom), and can be assumed to be removed from the description of the system by proper coordinate substitution. Nonholonomic constraints, on the contrary, do not reduce the number of degrees-of-freedom of the system, but rather reduce the number of independent coordinate velocities.

To describe in more detail contact constraints that are in effect in dextrous manipulation systems, consider a contact between the i-th phalanx and the object, occurring at time t at a point described in an inertial base frame B by the vector xi. A generic point on the surface of the phalanx will be described, in a frame  $C_i$  fixed on the phalanx, by the vector  $\mathbf{x}_i$ . Note that,  $\mathbf{x}_i \in \mathbb{R}^3$  is actually bounded to lie on the (assumed regular) surface Si of the link, and therefore can be regarded as a mapping  $f_{\mathbf{x}_i}: f_{\alpha_i} \in U_i \subset \mathbb{R}^2 \mapsto S_i \subset \mathbb{R}^3$ . The pair  $(U_i, f_{\mathbf{x}_i}(f_{\alpha_i}))$  is called a chart for (a portion of) the surface  $S_i$ , and the 2-vector  $I_{\alpha_i}$  is referred to as the point coordinates on the *i*-th link. Orthogonal coordinates can be chosen so that the associated metric tensor is diagonal. A normalized Gauss frame can be associated with each point on the surface chart that has the origin in the point and is fixed w.r.t

to the body so that its  $\zeta$  axis is aligned with the outward pointing normal, while the  $\chi$  and  $\xi$  axes span the tangent space. The orientation of the Gauss frame centered in x, w.r.t the C, frame can be expressed by a rotation matrix 'Ri. For notational compactness, a 6-vector 'ci composed of three components of  $f_{\mathbf{x}_i}$  and three orientation parameters  $f_{\theta_i}$  is often used to locally describe the position and orientation of Gauss frames. Similar considerations and definitions hold for the object surface.

Several types of contact models can be used to describe the interaction between the links and the object, among which the most useful are probably the point-contact-with-friction model (or "hard-finger"). the "soft-finger" model, and the complete-constraint model (or "very-soft-finger") [Salisbury and Roth, 1982] [Cutkosky, 1985]. In each case, the constraints consist in imposing that some components of the relative velocity between the Gauss frames that are associated with the contact point on each surface,

$$\mathbf{H}_{i}\left({}^{o}\dot{\mathbf{c}}_{i}-{}^{f}\dot{\mathbf{c}}_{i}\right)=0\tag{2}$$

where H<sub>i</sub> is a constant selection matrix. Being the two frames fixed on the object and the phalanx, respectively, their velocities can be expressed as a function of the velocities of the object and of the joints

$$f_{\dot{\mathbf{c}}_i} = \mathbf{J}_i(f_{\alpha_i, \, \mathbf{q}_i}) \, \dot{\mathbf{q}}. \tag{4}$$

Similar relationships hold for each contact point, and a single equation can be built to represent all constraints by properly juxtaposing vectors and block matrices to obtain

$$\mathbf{H}\mathbf{G}^T\dot{\mathbf{u}} - \mathbf{H}\mathbf{J}\dot{\mathbf{q}} = 0. \tag{5}$$

The matrix G is usually termed as the "grasp matrix", or "grip transform", while J is referred to as the hand Jacobian. A goal of the kinematic analysis of manipulation systems is to explicit the relationships between joint positions and object positions. If Ri and pi are the rotation matrix and the origin displacement of frame C, with respect to base, and R, and p, are analogous for the object frame, for a contact occurring at coordinates 'α on the finger and 'α on the object we can write the condition that the two points coincide as

$$\mathbf{R}_{\sigma} \, {}^{\sigma} \mathbf{x} ({}^{\sigma} \alpha) + \mathbf{p}_{\sigma} - \mathbf{R}_{i} \, {}^{i} \mathbf{x} ({}^{i} \alpha) - \mathbf{p}_{i} = 0 \tag{6}$$

Also, the condition that the normals at the contact point must coincide can be expressed as

$$\mathbf{R}_{o} \, {}^{o}\zeta({}^{o}\alpha) + \mathbf{R}_{i} \, {}^{i}\zeta({}^{i}\alpha) = 0. \tag{7}$$

A pair (R, p) (in mathematical terms, an element of SE(3)), completely defines the configuration of a rigid body in 3-space. Not all configurations of the phalanx relative to the object will cause contact: let W C SE(3) be the set of relative configurations that are in contact. Eq.(6) and eq.(7), grouped in an equation of the type h(°a,'a,° R,° p,'R,'p) together implicitly define a relationship between the relative configurations in W and  $f_{\alpha}$ ,  $\sigma_{\alpha}$ , which can be shown to be smooth and locally invertible by the use of the implicit function theorem under certain hypotheses on the relative curvature of surfaces [Murray and Sastry, 1990]. Note that the explicit derivation of this relationship is not possible in all but the simplest cases. Also, note that this result has been obtained without reference to the kinematic structure of the hand. If the dependence on q of the relative configurations are explicited via the manipulator forward kinematics, a bijection exists between the control inputs (joint positions) and contact coordinates. However, in whole-hand manipulation, the degrees-of-freedom of the hand are not sufficient to achieve arbitrary configurations of all the phalanges used for contacting the object (we will call such systems kinematic vally defective). Therefore, the phalanx configuration space is reduced to a subset  $Z \subset SE(3)$ . The existence of a nonvoid intersection set  $W \cap Z$  is a peculiar problem of defective manipulation systems, and one that has been investigated surprisingly little in the literature.

Another aspect of kinematics analysis is concerned with differential relationships between the variables. Corollaries of the implicit function theorem can be used to find the (explicit) relationship existing between the time derivatives of the contact coordinates and the velocities of the finger and of the object, which takes on the form

 $\begin{bmatrix} {}^{i}\dot{\alpha} \\ {}^{o}\dot{\alpha} \end{bmatrix} = \mathbf{B}({}^{i}\alpha, {}^{o}\alpha, \mathbf{u}, \mathbf{q}) \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{u}} \end{bmatrix}. \tag{8}$ 

where  $B(\cdot)$  is an usually involved nonlinear vector function depending on the geometry of the surfaces, and on the type of contact between the objects (see Montana [1988] for an explicit formula). Eq.(8) can be used to plan hand motions to achieve desired object trajectories. This problem is usually approached as an optimal nonlinear control problem, as discussed for instance by Murray and Sastry [1990]. It must be noted that the solution may result highly sensitive to modeling inaccuracies.

An important question in the differential kinematics analysis is: which object motions are possible starting from a given configuration, and to which joint motions do they correspond? This question can be easily answered if the mechanism under consideration is not defective. In fact, in this case the matrix HJ must be full rank, and we can write eq.(5) as

$$\dot{\mathbf{q}} = (\mathbf{H}\mathbf{J})^{+}\mathbf{H}\mathbf{G}^{T}\dot{\mathbf{u}} + (\mathbf{I} - (\mathbf{H}\mathbf{J})^{+}\mathbf{H}\mathbf{J})\mathbf{y},$$
(9)

where  $(\mathbf{HJ})^+$  is the Moore-Penrose pseudo-inverse of HJ, and y is a free vector that parameterizes the homogeneous (redundant) part of the solution. A whole-hand manipulation system, however, generally contains kinematically defective members (such as inner phalanges or the palm), and therefore HJ is not full row rank. The relationship between  $\dot{\mathbf{u}}$  and  $\dot{\mathbf{q}}$  for general manipulation systems (including whole-arm) has been considered by Bicchi and Mclchiorri [1992], where it was shown that there exist three vectors  $\nu_1, \nu_2$ , and  $\nu_3$  (whose dimensions vary with the problem at hand) such that every possible pair of object velocity  $\dot{\mathbf{u}}$  and joint velocity  $\dot{\mathbf{q}}$  that comply with the kinematic and contact constraints of the hand system can be written as

$$\dot{\mathbf{u}} = \mathbf{U}_o \nu_1 + \mathbf{U}_p \nu_2 \\ \dot{\mathbf{q}} = \mathbf{Q}_p \nu_2 + \mathbf{Q}_o \nu_3$$
 (10)

The columns of  $U_p$  and those of  $Q_p$  form a basis of the subspaces of compatible object and joint velocities, respectively. Any object motion described by the coordinate vector  $\nu_2$  in the image of  $U_p$  must correspond to a joint motion with the same coordinates in the basis  $Q_p$ . The images of  $Q_o$  and  $U_o$  represent the subspaces of redundant joint velocities and under-actuated object velocities, respectively.

Note that the matrices appearing on the right hand side of eq.(10) are functions of the position of the contact point on the surfaces. Again, this may represent a major obstacle in obtaining explicit expressions for the joint motions that are required to perform a desired object motion. Notice also that, besides the analytical difficulties, in practice we often have the case that the geometry of the object is poorly known, if at all. The availability of contact sensors that are able to provide information on the position of the contact points on the phalanges is therefore necessary to attempt closed loop control of fine manipulation. In particular, if joint angles and contact points are sensed, eq.(10) can be used even without information on the geometry of surfaces to control the object motion about desired trajectories by using generalized resolved-rate control. Also, tactile sensor information can be used to locally estimate the curvature form of the object surface, as discussed by Montana [1986].

### 3 Force Distribution

The problem of controlling contact forces in a multiple manipulation system such as a hand, a pair of cooperating robot arms, or a legged vehicle, has been traditionally considered in the assumption that every single finger has full mobility in its task space. This assumption greatly simplifies the problem, by allowing to separately deal with the analysis of the distribution of grasp force among the contacts, and with the control of the joint torques that realize desirable contact forces.

Let for instance an object be grasped by means of n contacts and let the components of contact forces and moments on the object form a vector  $\mathbf{t} \in \Re^4$ . Consider the task of resisting an external force  $\mathbf{f} \in \Re^3$  and moment  $\mathbf{m} \in \Re^3$  applied upon the object (the task of steering an object along a desired trajectory is equivalent once the inertial load corresponding to the specified acceleration and velocity profile is determined). The force and moment balance equation for the object can be written in matrix

notation as

$$\mathbf{w} = -\mathbf{G}\mathbf{t},\tag{11}$$

where  $\mathbf{w} = (\mathbf{f}^T \mathbf{m}^T)^T \in \Re^6$  is the so-called load "wrench". This equation has a solution in the hypothesis that  $\mathbf{w}$  belongs to the range space of  $\mathbf{G}$ , (i.e.,  $\mathbf{w} \in \mathcal{R}(\mathbf{G})$ ). In general, eq.(11) has more unknowns than equations, so that the solution is not unique. The general solution can be written as

$$\mathbf{t} = -\mathbf{G}^R \mathbf{w} + \mathbf{A}\mathbf{x}; \tag{12}$$

where  $\mathbf{G}^R$  is a right-inverse of the grasp matrix, and  $\mathbf{A} \in \mathfrak{R}^{t \times h}$  is a matrix whose columns form a basis of the nullspace of  $\mathbf{G}$ . The coefficient vector  $\mathbf{x} \in \mathfrak{R}^h$  parameterizes the homogeneous part of the solution eq.(12), which is usually called the "internal force": for any choice of  $\mathbf{x}$ , a vector of contact forces results that equilibrates the desired load. Although internal forces do not affect the overall dynamics of the object, their increase usually favours the robustness of the grasp against possible slippage. On the other hand, too high internal forces may be undesirable because of power consumption or possible damage to the object. Accordingly, an "optimal" set of internal forces can be defined as the one that is further away from violating these and other possible constraints. The techniques that can be applied to implement such optimal grasps (see e.g. [Nakamura, Nagai and Yoshikawa, 1989]; [Bicchi, 1992]) basically consist in defining a suitable cost function  $V(\mathbf{x})$  and finding its (constrained) extremum,  $\hat{\mathbf{x}}$ . The corresponding  $\hat{\mathbf{t}} = -\mathbf{G}^R \mathbf{w} + \mathbf{A}\hat{\mathbf{x}}$  is the optimal force distribution among contacts with respect to the criterion adopted in the design of V. Finally,  $\hat{\mathbf{t}}$  is applied by the fingers under some type of force control technique.

It should be noted that this last sentence tacitly relies upon a fundamental assumption, i.e. that any arbitrary distribution of contact forces t can be actively controlled by the robot. To discuss this assumption, consider the linear relationship between the contact forces on the fingers and the vector  $\tau \in \Re^q$  of joint actuator torques:

$$\tau = \mathbf{J}^T \mathbf{t}$$
. (13)

A robot system with  $q > \text{rank } (\mathbf{J}) = t$  is a "redundant" system, while if  $q \ge \text{rank } (\mathbf{J}) < t$ , the robot system is defective with respect to its task space dimension. For a redundant manipulator, a many-to-one mapping  $\mathbf{t}(\tau): \Re^q \to \Re^t$  can always be established which is onto  $\Re^t$ . For a non-redundant, non-defective (minimal) manipulator with  $q = \text{rank } (\mathbf{J}) = t$ , a one-to-one and onto mapping can be established between  $\Re^q$  and  $\Re^t$ . In both cases, arbitrary t's can be realized by suitably regulating joint torques. However, solution 12 can not in general be applied to defective manipulating systems, since there is no guarantee that the optimal contact forces can actually be realized by the robot. A simple self-explanatory example is shown in fig. 2. An issue of controllability therefore arises in considering grasping of objects by means of hands with fewer degrees of freedom than necessary to actuate all internal forces.

## 4 Dynamics and Control

The hand-object system consists of a constrained mechanical system, whose dynamical description can be derived using Lagrange's equations together with constraint equations. Consider first the dynamics of the hand and of the object separately:

$$\left(\frac{d}{dt}\frac{\partial L_h}{\partial \dot{\mathbf{q}}} - \frac{\partial L_h}{\partial \mathbf{q}}\right)^T = \mathbf{H}_h(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_h(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{V}_h(\mathbf{q}) = \tau$$
(14)

$$\left(\frac{d}{dt}\frac{\partial L_o}{\partial \dot{\mathbf{u}}} - \frac{\partial L_o}{\partial \mathbf{u}}\right)^T = \mathbf{H}_o(\mathbf{u})\ddot{\mathbf{u}} + \mathbf{C}_o(\mathbf{u}, \dot{\mathbf{u}})\dot{\mathbf{u}} + \mathbf{V}_o(\mathbf{u}) = \mathbf{w}, \tag{15}$$

where  $L_h$  and  $L_o$  are the hand and object Lagrangians, respectively, the  $\mathbf{H_i}(\cdot)$  are inertia matrices, the  $\mathbf{C}(\cdot, \cdot)$  terms include velocity-dependent forces, and the  $\mathbf{V}(\cdot)$  terms represent gravity and friction forces. These two equations are then attached by means of the velocity constraint eq.(5). Murray and Sastry [1990] discussed this dynamic problem in the hypothesis that the hand Jacobian (HJ in our notation) is full row rank, which fact allows to explicit the connected dynamics in terms of the undependent variables  $\mathbf{u}$  by using eq.(9).

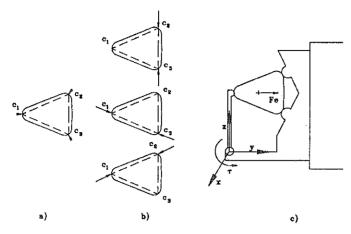


Figure 2: A simple example of grasp with a kinematically defective device. Contact forces pushing or pulling the object along the edges of the "grasp triangle" (fig.2-b) and their combinations are "internal" forces. However, if the grasp is to be realized by the simple single-joint gripper (fig.2-c), some combinations may not be feasible (for instance, opposing forces in the direction  $c_2 - c_3$  as in (fig.2-b, top).

In whole-hand manipulation, however, the hand kinematics may be defective and its Jacobian not full row rank. Thus, we rewrite the Lagrange equation for the whole system as

$$\left[\frac{d}{dt}\frac{\partial(L_h + L_o)}{\partial(\dot{\mathbf{q}}, \dot{\mathbf{u}})} - \frac{\partial(L_h + L_o)}{\partial(\mathbf{q}, \mathbf{u})} - (\boldsymbol{\tau}^T \mathbf{w}^T)\right] \begin{pmatrix} \delta \mathbf{q} \\ \delta \mathbf{u} \end{pmatrix} = 0$$
(16)

Transposing and using the notation introduced above, we have

$$(\delta \mathbf{q}^T \ \delta \mathbf{u}^T) \begin{pmatrix} \mathbf{H}_h(\mathbf{q}) \tilde{\mathbf{q}} + \mathbf{C}_h(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{V}_h(\mathbf{q}) - \tau \\ \mathbf{H}_o(\mathbf{u}) \ddot{\mathbf{u}} + \mathbf{C}_o(\mathbf{u}, \dot{\mathbf{u}}) \dot{\mathbf{u}} + \mathbf{V}_o(\mathbf{u}) - \mathbf{w} \end{pmatrix} = 0.$$
 (17)

If eq.(10) is now used to eliminate redundant generalized coordinates, the system dynamics can be rewritten as

$$\mathbf{Q}_o^{\mathrm{T}}(\mathbf{H}_h(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_h(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{V}_h(\mathbf{q}) - \tau) = 0; \tag{18}$$

$$\mathbf{U}_{o}^{T}(\mathbf{H}_{o}(\mathbf{u})\ddot{\mathbf{u}} + \mathbf{C}_{o}(\mathbf{u}, \dot{\mathbf{u}})\dot{\mathbf{u}} + \mathbf{Y}_{o}(\mathbf{u}) - \mathbf{w}) = 0; \tag{19}$$

$$\mathbf{Q}_{p}^{T}(\mathbf{H}_{h}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_{h}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{V}_{h}(\mathbf{q}) - \tau) + \mathbf{U}_{p}^{T}(\mathbf{H}_{o}(\mathbf{u})\ddot{\mathbf{u}} + \mathbf{C}_{o}(\mathbf{u}, \dot{\mathbf{u}})\dot{\mathbf{u}} + \mathbf{V}_{o}(\mathbf{u}) - \mathbf{w}) = 0.$$
(20)

To this set of vectorial differential equations, the algebraic constraints of eq.(10) are appended to form the system dynamics DAE. However, dynamics in this form are not manageable for control design and analysis. An explicit formulation of the dynamics can be obtained by taking the derivative of the constraint eq.(10) and substituting in the dynamics equation. Assuming  $Q_o = U_o = 0$  for simplicity, and letting  $\mu$  be the quasi-coordinate whose derivative is  $\nu_2$ , we obtain the dynamics of  $\mu$  in the form

$$\ddot{\mathbf{H}}(\mu)\ddot{\mu} + \tilde{\mathbf{C}}(\mu,\dot{\mu})\dot{\mu} + \tilde{\mathbf{V}}(\mu) = \tilde{\tau} + \tilde{\mathbf{w}},\tag{21}$$

where  $\tilde{\mathbf{H}} = \mathbf{Q}_p^T \mathbf{H}_h \mathbf{Q}_p + \mathbf{U}_p^T \mathbf{H}_o \mathbf{U}_p^T$ ;  $\tilde{\mathbf{C}} = \mathbf{Q}_p^T (\mathbf{C}_h \mathbf{Q}_p + \mathbf{H}_h \dot{\mathbf{Q}}_p) + \mathbf{U}_p^T (\mathbf{C}_o \mathbf{U}_p + \mathbf{H}_o \dot{\mathbf{U}}_p)$ ;  $\tilde{\mathbf{V}} = \mathbf{Q}_p^T \mathbf{V}_h + \mathbf{U}_p^T \mathbf{V}_o$ ;  $\tilde{\tau} = \mathbf{Q}_p^T \tau$ ; and  $\tilde{\mathbf{w}} = \mathbf{U}_p^T \mathbf{w}$ . Note that eq.(21) is in the canonical form of robot dynamics, and therefore is suitable for application of most control techniques proposed for the control of conventional robots, such as computed-torque methods. However, much of the structure of eq.(21) remains to be studied, especially from the point of view of planning object motions, to let alone problems in real-time evaluation of the rather messy terms in the above dynamics.

#### 5 Conclusions

The potentials of whole-hand manipulation for a more powerful and versatile robotic grasping seem to be promising enough to warrant deeper investigation of the new theoretical problems that are posed. In this paper, we emphasized some of these new problems. In particular, it has been shown that important restrictions affect the kinematics of these systems; that internal forces may result uncontrollable; and that the dynamics of whole-hand systems pose non-trivial problems in terms of the most suitable state representation for control purposes.

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