Intrinsic Contact Sensing for Soft Fingers

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Abstract

In this paper the basic mathematical relationships of "intrinsic" (or "force-based") contact sensing are discussed. While conventional tactile sensing devices are designed to provide information about local phenomena caused by contact (typically concerning the spatial distribution of normal pressures), intrinsic contact sensing detects a few global quantities related to the interactions of two bodies in contact. However, these quantities are very important from the point of view of manipulation control. The paper addresses the geometric-mathematical problem of detecting these quantities starting from force/torque measurements and from the geometric description of one of the contacting surfaces. Two methods for solving the intrinsic contact sensing problem are discussed. The first method is able to give exact results for contacts of the "hard finger" type, while it is shown to be only approximate for "soft finger" contacts: a formula for estimating the extent of such approximations is provided. A second, novel solution method is presented, which applies to soft fingers with ellipsoidal surface, and is capable of yielding exact solutions to the problem. Finally, some implementation issues and applications of intrinsic tactile sensing to fine manipulation operations are reviewed.

1 Introduction

Contact is the fundamental physical phenomenon on which manipulation relies. Contact is not only the means by which parts of the manipulator (most often its fingers) can exert forces on objects so as to control their motion, but also a source of sensorial information about manipulated objects. Therefore, robotics researchers have been devoting much attention in the recent past to replicating the capability of humans of sensing contact through tactile sensors.

Most attempts conducted so far used the human example not only as a performance target for their devices, but also as a model to follow in designing them. Thus, conventional contact sensing devices usually consist of many simple pressure transducers, arrayed on or in proximity of the surface of the robot finger, in much the same way that Pacini, Meissner tactile corpuscles are located in our skin. Although several such skin-like tactile sensors have been realized showing good results, there are some limitations inherent in this approach. Besides technological difficulties in building a high resolution sensor with the desired shape and material compliance, the high number (ranging typically from several tens to hundreds) of transducing elements of a skin-like sensor is an obstacle to its use in real time tasks.

Although the "tactile imaging" capability of skin-like sensors can be useful for some advanced perceptual tasks (e.g., for identifying features of an explored object which are smaller than the robot finger itself), the large majority of manipulation tasks do not require such sophisticated information. What is really needed in most cases is knowledge of some global aspects of contact, that is the overall effects of punctual actions exerted between the bodies in the contact zone: in short, where the finger is touching, how large the contact force is, and how it is directed. It can be observed that such simplified information is pretty much the same which is felt by a human being wearing gloves or thimbles for his/her work.

In this paper the application of the intrinsic contact sensing approach to soft fingers is examined. In section 2 an analysis of the model of soft finger contact is carried out, aimed at clarifying the assumptions that are made and to define the quantities that completely define a contact under this model. In particular, the concept of contact centroid is introduced in order to render the definition of "soft finger" contact more precise. In section 3 the problem of extracting the desired information from force/torque measurements is formulated. The hard finger solution of [1] is revisited in section 4. This solution is not theoretically correct if applied to soft fingers contacts. However, the method is shown to give approximate results, and an estimate of the extent of the approximation error is derived. In section 5 a novel method is presented, which applies to soft fingers with ellipsoidal surface, and is capable of yielding exact solutions to the problem. The solving algorithms are also given for some particular and limit cases of ellipsoidal surfaces, such as spheres, cylinders and planes.

Force-based contact sensors have been actually implemented [4, 5], and effectively employed in robotic hands. Although the main purpose of this paper is to provide the theoretical basis and the resolving formulas of intrinsic contact sensing for soft fingers, some implementation issues and applications of such sensors to fine manipulation operations are discussed in the final section.

2 The Soft Finger Contact Model

Consider the contact of two elastic bodies depicted in figure 1. Assume at first that the two bodies have non-conformal surfaces, so that, when brought into contact by a negligibly small force, they touch at a single point, . Let be taken as the origin of a cartesian reference frame , whose axis is chosen to coincide with the normal direction common to the two surfaces at . The plane is therefore tangent to the surfaces, and is referred to as the osculating plane [6]. Relative motions , along the tangential axes are commonly called slipping motions. A relative rotation , about the normal axis is termed slip, while a
motion $v_x$ is sometimes called approach. As bodies are pressed by a
finite intensity force, contact is made over an area $S$ of finite extension.
At each point $r$ of the contact area, bodies mutually exert tractions
$h = h(r) = (h_x, h_y, h_z)^T$. The $h_z$ component is usually referred to
as pressure, while $h_x$ and $h_y$ are the friction components. The overall
(resultant) contact force vector $p$ is defined as:
\[ p = \int_S h \, dS, \]
while the components of the resultant torque vector $q_a$ (with respect
to the $u_{xy}$ frame) are given by:
\[ q_{ax} = \int_S h_x y \, dS, \]
\[ q_{ay} = -\int_S h_y x \, dS, \]
\[ q_{az} = \int_S (h_x x - h_y y) \, dS. \]
It should be pointed out that the above relationships for the moment components are only valid for cases when the the contact area warping
is negligible; otherwise, contributions of friction components to rolling
resistance moments $q_{ax}$ and $q_{ay}$ should be accounted for. It is some-
times useful to consider also discrete contact points, through which a
force of finite value is transmitted. In this case, obvious modifications
should be accounted for. It is some-
times useful to consider also discrete contact points, through which a
force of finite value is transmitted. In this case, obvious modifications
apply to the integrals above.

According to the Hertz theory of contact mechanics (which is re-
stricted to frictionless, smooth surfaces and perfectly elastic solids),
the projection of the contact area $S$ on the osculating plane is an el-
ipse, symmetric with respect to $u$. In the following, however, different
hypotheses will be assumed. Friction forces will not be neglected; on
the contrary, emphasis will be put on their measurement and control,
which are fundamental in manipulation operations. No hypotheses will
be made about the pressure distribution over the contact area, nor the
contact area itself is supposed to have any simple shape or even to be
simply connected. This means that contacts occurring on several small
areas and/or points (as is the case when irregular surfaces are brought
into contact) will be allowed, as long as contact points are close enough.
In order to quantify this "close enough", let the contact convex hull be
defined as the smallest convex portion of the contacting surfaces that
encloses every contact point and/or area. Thus, we formulate the only
two hypotheses about contact that will be assumed in the following:

**Hyp.1:** contact pressures can only be directed towards
the inside of the bodies;

**Hyp.2:** the dimensions of the contact convex hull are
small compared to the radii of curvature of the surfaces in
the vicinity of the contact zone.

Figure 1: Initial contact between non-conformal surfaces.

The first assumption disregards adhesive forces through contact.
This is largely justified for almost every practical case.

The second hypothesis allows one to treat uniformly simple and
multiple point contacts, and some cases of contact between compliant
or conformal surfaces. For instance, the algorithms proposed in the
following can be applied to contacts between finite portions of planes.
Equivalent to the hyp.2 is the assumption that the contact convex hull
approximately lies on a plane (the contact plane, $e$). It can be noted
that the class of contacts considered is much wider than the point-
contact-with-friction model assumed in previous literature. The con-
tact convex hull $H$ is represented in figure 2 along with some small
areas $S_i$ where contact actually occurs.

Having dropped the Hertzian assumption about the symmetry of
the contact area, the choice of $e$ (the point where contact first occurred)
as the origin of the reference frame is somewhat abstract and no longer
convenient. On the other hand, it can be observed that in the original
proposition of this contact type [3, 7], soft fingers have been assumed
to transmit a force and a moment about the normal axis, without
specifying where in the contact area the force is applied and the normal
axis is defined. Therefore, the definition of a suitable reference frame
to characterize soft finger contacts appears to be necessary.

An expedient description of soft finger contact interactions can be
obtained as follows. It is well known that any set of forces (in particular
the set of contact forces exerted on each body) which is equivalent to
a resultant force $p$ applied at $u$ and a resultant moment $m_u$, is also
equivalent to a moment applied to the wrench axis of the set plus a moment
$q_u$ parallel to the same axis. It can be easily demonstrated that, in
general, given an arbitrary direction $l$, there exists a line such that $p$
applied at points on that line, and a moment $q_u$ parallel to the given
direction, form an equivalent set of forces. Using this result we define
the new origin of the soft finger contact reference frame as:

**Definition:** A contact centroid, $c$, is a point of a con-
tacting surface such that the set of contact forces is equiv-
alent to a resultant force $p$ applied at $c$ plus a moment $q$
normal to the surface at $c$.

The contact centroid has an important property: if hyp.1 and hyp.2
hold, the contact centroid lies in the contact convex hull. This can be
easily seen if the contact area is flat, since in this case the contact
centroid coincides with the centroid of the distribution over $S$ of the
normal pressures $h_z$, which are assumed to be compressive everywhere.
A similar property holds even in less strict hypotheses, allowing finite
warping of the contact area, but the proof will be omitted here.

This characteristic of the contact centroid is very meaningful for
contact sensing. In fact, if the position of the contact centroid can be
measured, and an estimate $\delta$ of the dimension of the contact area
can be reasonably assumed (based e.g on the compliance of the finger
material and on the the intensity of contact force), then we are assured

Figure 2: The contact convex hull and the centroid of contact.
that the point of actual contact furthest from the contact centroid is at a distance less or equal to $\delta$.

The soft finger reference frame will be defined as the orthonormal frame $\mathbf{c}_{\text{app}}$, centered in $\mathbf{c}$, whose $\mathbf{c}_x$ axis is parallel to the inward normal direction to the contacting surface in $\mathbf{c}$, and whose $\mathbf{c}_y$ axis is perpendicular to the resultant force $\mathbf{p}$. The $\mathbf{c}_{\text{app}}$ plane will be referred to as the contact plane $\pi$. In this reference frame, the set of contact forces can be described by their resultant $\mathbf{p}_r = (\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z)^T$ and by a torque $\mathbf{q}_r = (0, 0, \mathbf{q}_z)^T$. The $\mathbf{p}_r$ and $\mathbf{q}_r$ components of $\mathbf{p}_r$ are the normal and friction components of the contact force respectively. The only non-null component of $\mathbf{q}_r$, $\mathbf{q}_z$, is the spin resisting torque. The components of $\mathbf{p}$ and $\mathbf{q}_r$, along with the location of the contact centroid on the sensor surface, form a set of data which are sufficient to completely describe the global characteristics of a soft finger contact. The following sections will discuss how these quantities can be elicited from remote force/torque measurements.

3 Problem Formulation

An intrinsic contact sensor consists of a six-axis force/torque sensor and of a finger, whose surface is known. Let $\mathbf{B}_{\text{app}}$ be the reference frame attached to the sensor, and $\mathbf{f}$ and $\mathbf{m}$ the measured resultant force and moment in that frame, respectively. The choice of $\mathbf{B}_{\text{app}}$ is arbitrary, since simple relationships allow to express $\mathbf{f}$ and $\mathbf{m}$ in any other frame rigidly fixed to $\mathbf{B}_{\text{app}}$. The surface of the fingertip can be described by an implicit relationship as:

$$S(r) = 0,$$

where $r$ describes the generic point of 3-D space with respect to $\mathbf{B}$. The surface $S(r) = 0$ is supposed to have continuous first derivatives everywhere, so that a normal unit vector can be defined at every point on $S$ as:

$$\mathbf{n} = \frac{\nabla S(r)}{||\nabla S(r)||},$$

where $\nabla$ is the gradient operator.

The knowns and unknowns of the intrinsic contact sensing problem are summarized in figure 3. Force and moment balance equations in the reference frame $\mathbf{B}_{\text{app}}$ are written, in vector notation, as:

$$\begin{align*}
\mathbf{f} & = \mathbf{p}_r, \\
\mathbf{m} & = \mathbf{q}_r + \mathbf{c}_x \times \mathbf{p}_r,
\end{align*}$$

where every vector is expressed in the $\mathbf{B}_{\text{app}}$ frame, and the relative subscript has been omitted. Note that the contact force and torque in the sensor frame and in the above-described soft-finger reference frame are easily related as:

$$\begin{align*}
\mathbf{p}_r &= \mathbf{p}^T \mathbf{n} ; \\
\mathbf{q}_r &= \mathbf{q}^T \mathbf{n}.
\end{align*}$$

To force the contact centroid to lie on the fingertip surface, we impose that:

$$S(r) = 0.$$  

(3)

Finally, since $\mathbf{q}$ is parallel to $\mathbf{n}$, it holds:

$$\mathbf{n} \times \mathbf{q} = \frac{K}{2} \nabla S(r),$$

(4)

where $K$ is an unknown constant introduced for convenience.

Expanding equations 1 through 4 yields a non-linear system of ten equations in ten scalar unknowns, i.e. the nine components of $\mathbf{f}$, $\mathbf{m}$, $\mathbf{r}$ plus $K$. However, by simply substituting equation 1 and 4 in equation 2, the problem is reduced to four equations in four unknowns. Although the equations-unknowns balance is satisfied, the solution of the non-linear system of equations above is not trivial in general. The existence of a solution is not guaranteed for arbitrary $\mathbf{f}$, $\mathbf{m}$ and surface geometries. The uniqueness of the possible solutions cannot be assured unless further assumptions are made about the finger surface (this point will be discussed later). Moreover, a closed-form algorithm to solve the above stated problem is not expected to be available for all but the simplest surfaces. In the following, two methods for solving the above equations are discussed.

4 A Solution For Hard Fingers

In his seminal work [1], Salisbury proposed a method for contact sensing which amounts to a solution of equations 1 through 4 under tighter assumptions than we specified above. In particular, it is assumed that the resistance of friction to spinning relative motion is negligible, i.e. that $\mathbf{q} = 0$ (hard finger model of contact). In this case, it can be easily verified that the set of contact forces is equivalent to a pure force applied on the wrench axis. The wrench axis is described in the sensor frame $\mathbf{B}_{\text{app}}$, by the vectorial equation:

$$\mathbf{r} = \mathbf{r}_0 + \lambda \mathbf{f}$$

(5)

where

$$\begin{align*}
\mathbf{f} \times \mathbf{m} & = \mathbf{r}_0 - \frac{\mathbf{f} \times \mathbf{m}}{||\mathbf{f}||^2} \\
\mathbf{r}_0 & = \frac{||\mathbf{f}||^2}{||\mathbf{f}||^2} \mathbf{f}
\end{align*}$$

(6)

The wrench axis is a line through $\mathbf{r}_0$ and parallel to $\mathbf{f}$, parametrised by $\lambda$. The contact centroid will be found at the intersections of this line with the sensor surface, so that the problem is reduced to bare geometry, i.e. to solving the scalar equation $S(r) = 0$ in the only scalar unknown $\lambda$. In order to guarantee the uniqueness of the solution, one further hypothesis of convexity of the fingertip surface must be assumed. In fact, a line intersects a convex surface in at most two points, between which the contact centroid can be easily discriminated by enforcing the criterion $\mathbf{f}^T \mathbf{m}(\mathbf{c}) < 0$, which guarantees the resultant contact force to point inwardly at the sensor surface.

If this solving method (which will be called the wrench method) is applied to the more general problem of soft finger contact, where $\mathbf{q}$ is not null, the point so obtained will not coincide with the contact centroid; henceforth, the wrench method point will be denoted by the vector $\mathbf{c}_w$. It should be pointed out that the point $\mathbf{c}_w$ does not share the above discussed property of contact centroids, so that $\mathbf{c}_w$ could lie far away from the actual location of contact points/areas. Yet, the point $\mathbf{c}_w$ retains a valuable meaning in real conditions as an easy-to-compute approximation of the contact centroid. In order to give an estimate of the distance between $\mathbf{c}_w$ and the contact centroid $\mathbf{c}$, let $e$ represent the difference vector $e = \mathbf{c}_w - \mathbf{c}$ (see figure 4).

The two sets of forces $[\mathbf{f}, \mathbf{q}]$ applied at $\mathbf{c}_w$ and $[\mathbf{f}, \mathbf{q}_w]$ applied at $\mathbf{c}_w$ are both equivalent with the actual set of contact forces, hence they

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An Exact Solution for Soft Ellipsoidal Fingertips

As discussed above, the wrench axis method does not provide information about the spin torque exerted by friction on soft fingers. Moreover, this method gives approximations of the contact centroid which are unsatisfactory if high-friction and compliant materials are employed to build the finger. In this section, a different solution to the intrinsic contact sensing problem is proposed, which avoids these shortcomings.

In order to simplify the mathematics and to provide a closed-form algorithm, the fingertip surface will be constrained to belong to a specified class of surfaces, namely, to be a quadratic form of the type:

\[ S(r) = r^T A r - R^2 = 0, \]

where \( A \) is a constant coefficient matrix, and \( R \) is a scale factor used for convenience. Since the force/torque sensor reference frame \( B_{SP} \) can be moved arbitrarily in the space, we assume that \( A \) can be written in diagonal form as:

\[ A = \begin{pmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}. \]

In order to guarantee the uniqueness of the solutions that will be found, the surface specification must be restricted to convex portions of such a quadratic form; for instance, only one of the sheets of a double-sheet hyperboloid would be an appropriate sensor surface. Such caution is not necessary if only ellipsoids are considered: in such a case, the half-length of the ellipsoid axes are given by \( aR, bR \) and \( cR \), with

\[ 0 \leq 1/\alpha \leq 1, 0 \leq 1/\beta \leq 1, 0 \leq 1/\gamma \leq 1. \]

The choice of this class of surfaces is justified by several reasons:

- Quadratic forms can locally approximate, up to the second order, any continuous surface;
- Some very interesting surfaces (e.g., spheres, cylinders, and planes) can be regarded as particular cases of ellipsoids;
- This assumption is standard in contact mechanics (e.g., Hertzian theory of elastic contact), so that formulas providing estimates of the dimensions of the actual contact area (see section 2) can be easily applied.

Substituting equation 11 into equation 4 leads to:

\[ n = \| A^T c \| \times q = K A^T c, \]

and substituting this and equation 1 in equation 2, we obtain:

\[ m = K A^T c + e \times f. \]

Equation 11 and equation 13 form a system of four non-linear equations and four unknowns which can be rewritten in the following form:

\[ T c = m, \]

\[ c^T A c = R^2, \]

where \( T = \Gamma(K) \) is a \( 3 \times 3 \) matrix whose elements are functions of \( K \) and of the measured force components \( f_1, f_2 \) and \( f_3 \):

\[ \Gamma(K) = \begin{pmatrix} K/\alpha^2 & f_3 & -f_2 \\ -f_3 & K/\beta^2 & f_1 \\ f_2 & -f_1 & K/\gamma^2 \end{pmatrix}. \]

The determinant of \( \Gamma(K) \) is given by:

\[ \det \Gamma(K) = K(D^2) + \| A \|^2, \]

where \( D = \det A \). The matrix \( \Gamma(K) \) is singular for \( K = 0 \), i.e. when the local torque \( q \) is null. In this case, as we showed before, the contact centroid position can be exactly determined by the wrench method. The value of the parameter \( \lambda \) in equation 5 corresponding to the intersection of the wrench axis with the ellipsoid surface is given by:

\[ \lambda = -r^T r - \sqrt{(r^T r)^2 - \| r \|^2 \| c \|^2 - R^2}, \]
where \( f' = A f \) and \( r_r' = A r_0 \) (recall the definition of \( r_0 \) in equation 6).

The singularity of \( \Gamma(K) \) can be checked out directly from force/torque measurements by means of the simple equivalent relationship \( f^m = 0 \). Note also that this equation has zero likelihood of being verified exactly, if real (noisy) measurements are considered. In any other case, \( K \) is not null, and \( \Gamma(K) \) has an inverse \( \Gamma^{-1}(K) \) such that, solving equation 14 for \( c \), we obtain:

\[
    c = \Gamma^{-1}m = \frac{1}{\det K} K^{-1}(K^2 D^2 A^{-2} m + K(A^2 f' \times m) \times m + (f^m m) f').
\]  

(16)

By substituting this into equation 15, a scalar equation in the only unknown \( K \) is obtained as:

\[
    c^T A^2 c = R^2 = \frac{K^4 D^2 R^2 + K^2 |\hat{\Delta}A|^2 - D^2 |\hat{\Delta}A|^2}{K^2 (K^3 D^3 A^3 f')},
\]

(17)

After some simple if tedious algebra, it can be verified that:

\[
    \|A(A^2 f' \times m)\|^2 = D^2 |\hat{\Delta}A|^2 |\hat{\Delta}A|^2 - (f^m m)^2,
\]

so that equation 17 can be simplified in a biquadratic equation as:

\[
    K^4 D^2 R^2 + K^2 |\hat{\Delta}A|^2 - D^2 |\hat{\Delta}A|^2 - (f^m m)^2 = 0.
\]

Only one of the four possible \( K \) solutions to this equation is real and consistent with the hypothesis of non adhesive contact, and is given by:

\[
    K = \frac{-\operatorname{sign}(f^m m)}{\sqrt{3} D R} \sqrt{\sigma + \sqrt{\sigma^2 + 4 D^2 R^2 (f^m m)^2}},
\]

where:

\[
    \sigma = D^2 |\hat{\Delta}A|^2 - R^2 |\hat{\Delta}A|^2,
\]

\[
    \operatorname{sign}(\sigma) = \begin{cases} 
    -1, & \text{for } \sigma < 0 \\
    0, & \text{for } \sigma = 0 \\
    1, & \text{for } \sigma > 0
    \end{cases}
\]

By substituting back equation 18 into equation 16 and equation 12 gives the complete solution for \( c \) and \( q \), respectively.

5.1 Particular and Limit Cases

Equations 16 and 18 have simpler forms in some particular cases of practical importance:

5.1.1 Sphere

For a spherical sensor surface of radius \( R \) centered at the origin of the force/torque reference frame \( B_{sens} \), the \( A \) matrix equals the identity matrix \( I_3 \), and \( D = 1 \). Hence we have:

\[
    K = -\frac{\operatorname{sign}(f^m m)}{\sqrt{2} R} \sqrt{\sigma' + \sqrt{\sigma'^2 + 4 R^2 (f^m m)^2}},
\]

where \( \sigma' = |m|^2 - R^2 |m|^3 \).

The contact centroid position on the sphere, for \( K \) nonzero, is given by:

\[
    c = \frac{1}{K (R^2 + |f|^2)} [K^2 m + K f' \times m + (f^m m) f'].
\]

The solution for the particular case \( K = 0 \) is given by equation 5 when the value is substituted:

\[
    \lambda = \pm \sqrt{R^2 - \frac{|f^m m|^2}{|f|^2} \frac{1}{|f|^2}}.
\]

5.1.2 Cylinder

Consider for example a cylinder having the axis parallel to the \( B_s \) axis of the sensor frame, and circular cross section of radius \( R \). Such surface can be described as the limit case of an ellipsoid with characteristic matrix given by:

\[
    A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}
\]

for \( \gamma \to \infty \). Applying the same limit to equation 18, we have:

\[
    K = \frac{-f^m m}{\sqrt{R^2 |f|^2 - |m|^2 |f|^2}}
\]

where \( f^2 = (f_1, f_2, 0)^T \) is the component of \( f \) normal to the cylinder axis, and \( m^2 = (0, 0, m_3)^T \) is the component of \( m \) parallel to the same axis. If \( K = 0 \), the wrench method (equation 5) should be applied. Otherwise, the contact centroid on the cylindrical surface of the fingertip is given by:

\[
    c = \frac{1}{K |f|^2} [R m^2 + K f' \times m + (f^m m) f']
\]

5.1.3 Planes

An ellipsoid with matrix \( A \) of the form

\[
    A = \begin{pmatrix} 1/\gamma & 0 & 0 \\ 0 & 1/\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

degenerates, for \( \gamma \to \infty \), in a couple of parallel planes perpendicular to the \( B_s \) axis, at a distance \( \pm R \) from \( B \). If \( f^m = (0, 0, f_3)^T \) is the contact force component parallel the \( B_s \) axis, equations 18 and 16 become:

\[
    K = \frac{-f^m m}{R |f|^2}
\]

and

\[
    c = \frac{1}{|f|^2} [R m + f |f|^2].
\]

It is observed that the latter formula holds even with \( K = 0 \).

6 Discussion

One of the main points addressed in this paper is the definition of contact centroid for soft finger contacts, and the derivation of an algorithm for calculating its position starting from remote force/torque measurements. The interest of the contact centroid for characterizing soft fingers contacts follows from its property of being located inside the convex hull that encloses every actual point of contact. To illustrate this, a simple numerical example will be worked out. Assume that the real pattern of contact on the surface of a spherical sensor is comprised of only four points \( c_1 \ldots c_4 \), located on top of the sphere as shown in figure 5, such that their projections on the horizontal plane form a rhombus with semi-diagonal \( \delta \) and \( \theta \). Let the local contact forces exerted at these points be \( h_1 = (-h_2, h_2, 0) \), \( h_3 = (-h_4, h_4, -1) \), \( h_3 = (h_4, h_4, 0) \), and \( h_4 = (h_4, h_4, -1) \), respectively. Table 1 gives the \( x \)-coordinates of the contact centroid \( c \) (calculated through the algorithm proposed in section 5) and of the wrench method point \( c_w \) (section 4) corresponding to different values of \( \delta \) and \( \theta \). As it can be seen, the two results diverge as the distance \( \delta \) and the friction force \( h_f \) increase. It can be noted that, for \( \delta \to 0 \), the contact tends to a point-contact-with-friction model, while for small \( \delta \) the soft-finger model assumptions are verified. Note also that, although for large values of \( \beta \) the hypothesis 2 of section 2 is not satisfied, the contact centroid retains the characteristic of remaining inside the contact points, while the point obtained by the wrench method do not.
The exploitation of the information provided by sensors such as those described in this paper can be expected to allow improvements in many areas of fine manipulation control, since they provide direct, prompt and reliable feedback of fundamental contact characteristics. For instance, intrinsic contact sensors could be profitably used to improve the accuracy of the control of micro-motions of manipulated objects or tools, especially in the presence of slipping and/or rolling contacts.

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The other important advantage of the proposed algorithm is that the spin resisting torque originated from friction forces is calculated, without adding any new measurement.

Devices based on the force-based contact sensing approach have been actually implemented, and effectively employed in robotic hands. For a discussion on the realisation of force/torque sensors on small size robot fingertips, see [4, 2]. The approach followed in the latter paper consists in the application of optimal design techniques to miniaturized force/torque sensors, and is more thoroughly discussed in [8].

The applications of intrinsic contact sensors to robotic manipulation are various, and several have already been experimentally verified. Although it is not possible to detail these applications here, they will be cited for reference:

- The exploration of unknown objects by probing with an intrinsic tactile sensor, and the reconstruction of their surface profile has been described by [4], and later by [5]. Both authors employed the wrench method algorithm. In [9] and [10], the exploration has been performed using the more precise soft finger method, and an hybrid control scheme, which allowed continuous control of the normal component of contact force.
- The capability of intrinsic contact sensors to evaluate the friction components of the contact force and the spin torque (which is unique among other available sensing devices), has been used to measure the coefficients of friction of various objects [9]. This information in turn has been used to discriminate objects on the basis of their apparent friction, and to plan subsequent slippage-safe operations of the hand.
- A real-time control method for augmenting the stability of the grasp of unmodeled objects against slippage has been discussed and demonstrated (in a rather small robotic hand) in [11]. The ability of the algorithm described in this paper to provide a measurement of the spin resisting torque is instrumental in detecting and reacting to spin slippage danger, whose importance is often underestimated in manipulation planning and control.

### Table 1: Position of the contact centroid c and of the approximated (wrench method) point c, for different values of δ and h₂.

<table>
<thead>
<tr>
<th>δ</th>
<th>0.25</th>
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<th>0.5</th>
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<tbody>
<tr>
<td>h₂</td>
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<td>0.1</td>
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<tr>
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<tr>
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<tr>
<td>cₜ</td>
<td>0.000</td>
<td>0.000</td>
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</tr>
</tbody>
</table>

Figure 5: A simple contact pattern used as an example.